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Retailer-driven closed-loop supply chains with product remanufacturing

Jie Li
Iowa State University

Retailer-driven closed-loop supply chains with product remanufacturing

by

Jie Li

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Major: Industrial Engineering

Program of Study Committee:
Jo Min, Major Professor
Sarah Ryan
Sigurdur Olafsson
Mervyn Marasinghe
Frank Montabon

Iowa State University

Ames, Iowa

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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ABSTRACT

In this paper, we study a closed-loop supply chain (CLSC) consisting of a manufacturer and a retailer of a single product with numerous end product customers. The key feature differentiating this paper from the extant literature in CLSC is that our model is retailer-driven while the traditional CLSC models have been manufacturer-driven. This study is motivated by the emergence of super retailers such as Wal-Mart. Under these circumstances, we show how the retailer leads and the manufacturer follows in determining the wholesale and retail prices in the forward flow and the used product collecting and return prices in the reverse flow. In particular, we show via numerical examples a loss leader pricing behavior of the retailer who intentionally loses money for each unit returned in the reverse flow so as to make the maximum total profit considering both forward and reverse flows. Next, we investigate a centrally coordinated CLSC of the retailer and the manufacturer as well as the manufacturer-driven model, and compare the price and profit implications with the retailer-driven model. Also, we examine various ramifications of the retailer-driven model with respect to the coordination mechanisms, policy implications, and third-party logistics.

CHAPTER 1. INTRODUCTION

1.1 Supply Chain Power

There have been numerous papers in the manufacturer-retailer supply chain literature analyzing economic and managerial aspects based on production-oriented perspectives. In such papers, the implicit assumption is that products are made by manufacturers with some market power and are sold ultimately to final consumers. The manufacturer often leads the supply chain by deciding how many units to produce and at what price to sell. The retailer is often of less importance in terms of value-addition in the supply chain, and is treated as an intermediary between the manufacturer and the end product consumers (see e.g., Dawson, 2000).

However, the recent years have witnessed dramatic changes in the supply power structure and the role of the retailer in the supply chain. First, the economy has shifted into services-oriented systems. In such systems, part of the service is the product made by the manufacturer but part of it is made by the retailer, often at the store. What the consumer buys is not only the product from the manufacturer, but also all kinds of supportive and shopping service from the retailer, e.g., the delivery and setup of new electronics, the length of the queue at the pay point. Even in case that the manufacturer determines what to produce and sell, the retailer still produces the "experience" which the consumer buys. In many cases, the retailer has better knowledge of the demand of the consumer and seeks to procure (get made or buy from the manufacturer) the components of the service that the consumer wants. Therefore the retailer has become the central player in the supply chain.

Second, we have seen the rise of the "super retailers". With the help of the information technology, more effectible logistics networks and the globalization of the economy, these super retailers emerged and are able to seek competitive manufacturers worldwide. For example, Wal-mart has exerted tremendous supply chain power over their suppliers in the last decade.

Although in some monopolistic or oligopolistic markets the manufacturer is still able to charge above competitive prices, Gilo (1999) argues that the conventional wisdom that the manufacturer in noncompetitive markets has power to charge supra-competitive price is not necessarily correct. The bargaining power has at least partially, if not all, shifted to the retailer in many supply chains.

1.2 Remanufacturing

The importance of product remanufacturing has been widely recognized in the literature and in practice. In the US, the remanufacturing industry has become a $53 billion industry (Lund, 1998) and employed 480,000 people per year according to National Center for Remanufacturing and Resource Recovery (NC3R, 2005).

However, many of the remanufacturing systems in the industry are waste stream systems (Guide and Van Wassenhove, 2001) in which efforts are placed to diverting discarded products from landfills by making producers responsible for the collection and reuse of their products. The feature of the waste stream system is that the returns are random and based on regulations or citizenship; the producers passively accept all product returns from the waste stream and manage to minimize the disposing and related operating cost.

The counterpart of the waste stream system is the market-driven system (Guide and Van Wassenhove, 2001), which has attracted more attention in recent years. In such systems, the remanufacturing practice is profitable and the end-users are motivated to return end-of-life products by financial incentives, such as cash, credit toward a new unit, etc. The distinction of the market-driven system is the manufacturer's active control over the product returns.

For example, Xerox Corporation provides prepaid mailboxes so that customers can return their used copy or print cartridges for free; to further motivate product returns in Australia, Xerox credits A$10 (US$7) for each cartridge returned (Kerr, 2000). Hewlett-Packard (HP) recently had a Trade-In program, which gives up to $400 mail-in rebate to the customers who purchase a new printer and return an old one directly to HP (http://www.hp.com/). Another example is Staples, a major office product retailer store. Staples recently offered a $3 coupon for each returned cartridge (http://www.rechargermag.com/article.asp?id=200511040). The returned cartridges are remanufactured into Staples-brand cartridges or recycled. "Customers have been aware of Staples as a destination for toner and ink for a while now," Owen Davis of Staples said, 'We have made recycling available for a number of years, but this new program will hopefully encourage those who had previously not considered recycling their cartridges to do so". In addition, Guide and Van Wassenhove (2001) provides a case study of how ReCellular, Inc., a cell phone remanufacturer, transforms from a waste stream system to a market-driven system by providing a table of prices for specific cell phones with specific levels of quality.

Recently, there is one more driving force behind the market-driven remanufacturing system, i.e., the green design or design for reuse concept which aims to incorporate the idea of product return and easy-disassembly into new product design. In such systems, how to effectively control the product returns and how to interact with the collecting party is even more strategically crucial to the remanufacturer.

1.3 Research Questions

There is literature addressing the super retailer and the remanufacturing practice separately. However, there are few papers in the extant literature addressing both issues in a model. Motivated by this fact, the purpose of the research is to investigate the pricing strategies of a two-echelon market-driven closed-loop remanufacturing supply chain with the retailer as the Stackelberg leader. To provide comparison, we also discuss the centrally coordinated supply chain and the manufacturer-driven supply chain. More specifically, we seek answers to the following questions:

(1) How to set the prices in the market-driven remanufacturing supply chain, and how are the prices affected by the supply chain power structure?

(2) How is the profit of each party in the supply chain affected by the supply chain power structure?

(3) How the model can be used in the decision making?

We analyze the model in a steady-state setting. Key findings of the model show that the Stackelberg leader in the supply chain gives up the profit from the reverse flow and makes profit from the forward flow only. This is an interesting finding because this is a phenomenon very similar to the "loss leader pricing" policy in the economics literature. Loss leader pricing is "a pricing strategy in which retailers set very low prices, sometimes below cost, for some products to lure customers into stores" (Hess and Gerstner, 1987). The idea is that by using loss leader pricing policy, while retailers lose profit in these products, they could generate higher profits from other products. In this research, we observe that the Stackelberg leader in the supply chain adopt the loss leader pricing to the reverse flow.

In a broad sense, this research contributes to our understanding of the pricing strategies of the basic two-echelon market-driven closed-loop remanufacturing supply chain with different supply chain power structures.

1.4 Organization of the Dissertation

The dissertation is organized as follows. Literature review is given in Chapter 2. Chapter 3 discusses the formulation and solution of the two-echelon retailer-driven supply chain model, specifically in the linear demand and return rate function form. In Chapter 4 we provide numerical analysis of the retailer-driven supply chain model in the linear function form and show the loss leader pricing policy used by the retailer. Chapter 5 presents a limited optimality analysis to help gain an intuitive explanation of loss leader pricing policy in the retailer-driven supply chain model. Chapter 6 studies the centrally coordinated supply chain and manufacturer-driven supply chain. These two supply chain models have been studied in a few literatures, but we provide the models with different assumptions and derive the conclusion in a more general form. In Chapter 7 we present some discussion about the comparison of the models, the supply chain coordination issues, the policy implications and a third party collecting model extended from the basic model. Summary and future work is given in Chapter 8. To make the main contents concise and focused, most of the mathematical proof is moved to Appendix.

1.5 Definitions of Terms

This is a list of the basic terms used in the paper.

cm : the unit cost of manufacturing from raw materials.

cr : the unit cost of remanufacturing from a returned product.

$\Delta=cm-cr>0$: the direct savings from remanufacturing for the manufacturer.

w : wholesale price charged by the manufacturer to the retailer.

p : retail price charged by the retailer to the customer.

$\beta=p/w$: the forward flow price multiplier.

b : the unit return price paid by the manufacturer to the retailer for a returned product.

c : the unit collecting price paid by the retailer to the customer for a returned product.

$\lambda=c/b$: the reverse flow price multiplier.

$D(p)$: the demand function of the product.

$r(c)$: the return rate function of the used product, $r(c)\in[0, 1]$.

Π_R^M, Π_M^M, Π_C^M : Manufacturer's profit in the retailer-driven supply chain, in the manufacturer-driven supply chain, and in the centrally coordinated supply chain respectively.

Π_{R}^{R}, Π_{M}^{R}, Π_{C}^{R} : Retailer's profit in the retailer-driven supply chain, in the manufacturer-driven supply chain, and in the centrally coordinated supply chain respectively.

Π_{RDMC}^{R} : Retailer's profit in the Retailer-driven Manufacturer-collecting model.

$b_{RDMC}^{*} = c_{RDMC}^{*}$: The unit price paid by the manufacturer (collector) to the customer for returning a used product in the Retailer-driven Manufacturer-collecting model.

b_{R}^{*}, c_{R}^{*}, λ_{R}^{*} : The optimal return price, optimal collecting price and optimal reverse flow price multiplier respectively in the retailer-driven (retailer-collecting) model.

Π^{3P} : The profit of the 3rd party in the third party model

CHAPTER 2. LITERATURE

In this section, we present a review of several relevant streams of previous work in supply chain management and remanufacturing.

One stream of literature studies the supply chain power structure and its impact on the supply chain performance. Lau and Lau (2005) summarizes a list of the supply chain gaming processes commonly found in practice. Their research found that each of the gaming processes in the list appears to be no less plausible than the others in the practice. To model a super retailer as the Stackelberg leader in the supply chain, they found two basic approaches: a dollar amount markup and a percentage markup: In the dollar amount markup, the retailer declares a fixed dollar amount β and sets the retail price as $p = w + \beta$ (where w is the wholesale price and p is the retail price). In the percentage markup, the retailer declares a fixed percentage amount β and sets the retail price as $p = w\beta$. A similar modeling approach could be found in Ertek and Griffin (2002), which studies the supplier- and buyer-driven supply chains in a two stage supply chain with the forward flow only. In their paper, the buyer has dominant power in the buyer-driven supply chain and acts first to declare a non-negative mark-up α and non-negative price multiplier β and states that she will set price $p = \alpha + \beta w$, where w is the supplier's price. Their research finds that it is optimal for the buyer to set the mark-up α to zero and use only a multiplier β. Our model will apply a similar approach as above. Other papers studying the supply chain power include Kohli and Park (1989), Grant (1999), Munson et al. (1999), Messinger and Narasimhan (1995), Dawson (2000), etc.

A growing literature in operations management addresses remanufacturing and reverse logistics issue. Fleischmann et al.(1997) and Guide et al.(2000) provide comprehensive reviews of remanufacturing research. The three categories of remanufacturing issues identified by these reviews are production planning and control (e.g., Souza et al. 2002), inventory control (e.g., Toktay et al. 2000) and material planning (e.g., Ferrer and Whybark, 2001; Inderfurth, 2004). The common underlying assumption in these papers is that the decisions are made by a central decision maker.

Many papers in the traditional operations management and marketing area address the decentralized decision process and coordination issue in the supply chain. Thomas and Griffin (1996) provides a review of models of coordination mechanisms in the supply chain to reduce operating cost. Jeuland and Shugan (1983) and McGuire and Staelin(1983), from the marketing point of view, analyze the pricing issue of each party in the supply chain. The common feature of these papers is that they only look at the forward flow of product in the supply chain.

There are a few papers that combine the above two streams and address the decentralized decision process and coordination issue in a supply chain with both forward and reverse flows. Savaskan et al. (2004), by assuming that the manufacturer has sufficient supply chain power and acts as a Stackelberg leader, studies the problem of choosing the appropriate reverse supply chain structure for the collection of used products from customers. Their research demonstrates that the retailer is the most effective undertaker of returned product collection activity for the manufacturer. Majumder and Groenevelt (2001) presents a competition model in remanufacturing from strategic perspective, and suggests that incentive should be given to OEM manufacturer to decrease remanufacturing cost or increase product returns.

Finally, there is a growing research interest in the decision models for the used product acquisition. Instead of assuming that the product returns are an exogenous process and the remanufacturer passively accepts product returns, Guide and Van Wassenhove (2001) provides a framework of the market-driven product acquisition system to effectively control product returns, especially product quality. Guide et al. (2003) also provides a model of the acquisition management.

An interesting finding of this research coincides with a phenomenon called the “loss leader pricing” policy in the economics literature. Loss leader pricing is also referred to as “featuring” in some literature (e.g., Nelson and Hike 1986). Hess and Gerstner (1987) provides the rationale of loss leader pricing policy and studies the effects of such pricing policy on the retailer’s profit and market outcomes, given the retailer could accurately predict demand. Other literature regarding loss leader pricing includes Busch and Houston (1985) and DeGraba (2006). Even through our model does not set out to find an optimal loss leader pricing strategy, the results from profit maximization does lead to such pricing phenomenon.

Our research draws on and contributes to the above streams by developing a two-echelon market-driven closed-loop remanufacturing supply chain. Different scenarios of the supply chain power structure and the market-driven behavior of the product returns are integrated into the model.

The relevant literature is summarized in the following Table 1.

Table 1. Literature Review

<table>
<tr><td>Supply Chain Power</td><td colspan="2">Lau and Lau (2005): dollar amount and percentage markup retailer
Ertek and Griffin (2002): mixed markup retailer
Other: Kohli and Park (1989), Munson et al. (1999), Grant (1999), Messinger and Narasimhan (1995), Dawson (2000)</td></tr>
<tr><td>Remanufacturing</td><td>Review
Fleischmann et al.(1997)
Guide et al.(2000)</td><td>Production Planning: Souza et al. (2002)
Inventory Control: Toktay el al. (2000)
Material Planning: Ferrer and Whybark (2001), Inderfurth (2004)</td></tr>
<tr><td>Traditional Supply Chain Management (SCM)</td><td colspan="2">Thomas and Griffin (1996); Jeuland and Shugan (1983)McGuire and Staelin(1983), etc.</td></tr>
<tr><td>SCM with Remanufacturing</td><td colspan="2">Savaskan et al. (2004), Majumder and Groenevelt (2001)</td></tr>
<tr><td>Used Product Acquisition (Market-driven)</td><td colspan="2">Guide and Van Wassenhove (2001), Guide et al. (2003)</td></tr>
<tr><td>Loss Leader Pricing</td><td colspan="2">Hess and Gerstner (1987), Nelson, P.B. and Hike(1986), Busch and Houston(1985), DeGraba(2006).</td></tr>
</table>

CHAPTER 3. RETAILER-DRIVEN MODEL

In this chapter, we elaborate the retailer-driven closed-loop supply chain model. Our primary focus is to investigate the retailer-driven model because (1) there is an emerging trend that the retailer has more market power and leadership than the manufacturer (Dawson, 2000); (2) there are few papers in the extant literature addressing the retailer-driven scenario (cf. there are numerous papers addressing the manufacturer-driven scenario).

The organization of the chapter is: section 3.1 is assumption and notation, followed by the section 3.2 problem formulation. In section 3.3, we introduce the linear demand and linear return rate functions. Section 3.4 gives an intermediate solution of the model with the linear function form.

3.1 Assumptions and Notations

The retailer-driven two-echelon closed-loop supply chain with product remanufacturing is illustrated in Figure 1. The manufacturer manufactures a new product from raw materials with unit cost of *cm*, or remanufactures a returned product into a new one with unit cost of *cr*. We assume there is no distinction between the manufactured and remanufactured products (see Kerr 2000 for an example of photocopier) and $\Delta = cm - cr > 0$, i.e., the remanufacturing is potentially profitable (Δ is referred as direct savings from remanufacturing in this paper).

We also assume that the manufacturer could control the unit wholesale price of the new product ($w>0$) and the unit return price of the returned product from the retailer

($b>0$). The retailer-driven means that the retailer has sufficient supply chain power over the manufacturer to act as a Stackelberg leader. As in many literatures (see e.g., Lau and Lau, 2005; Ertek and Griffin, 2002), we assume that the Stackelberg gaming process is that the retailer takes an active role and declares two price multipliers ($\beta>0$ and $\lambda>0$) and states that the retail price p and collecting price c will be set as $p=\beta w$ and $c=\lambda b$. The retailer could predict the manufacturer's reaction (setting of w and b) given her action (setting of β and λ). By taking into account the manufacturer's reaction, the retailer tries to obtain her maximal profit by choosing the optimal β and λ. The customer is paid c for a returned product.

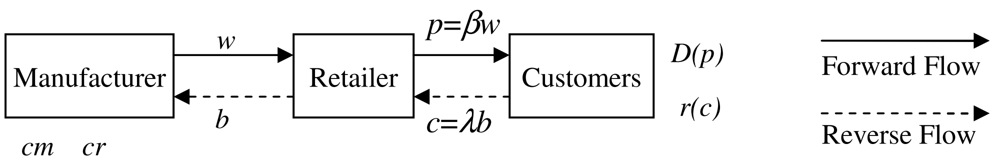


Figure 1. Closed-loop Two-echelon Supply Chain with Remanufacturing

The demand by the customers for the product is $D(p)$. The reverse flow is characterized by the return rate of the used product from the customers, $r(c)$, which is a function of the collecting price c. The total amount of the returned product is $D(p)r(c)$.

We note that our model is a generalization of the traditional economic model with the forward flow only (see e.g., Varian 1992). Specifically when remanufacturing is not profitable ($\Delta\leq 0$) or when the customers never return any product ($r(c)\equiv 0$), the reverse flow would not exist and our model is degenerated into the traditional forward-only model.

We also have the following assumptions in our model that are essential to answer our research questions:

(1) The model is considered in a steady-state setting;

(2) Both the manufacturer and the retailer have access to the supply chain information consisting of the manufacturing cost, remanufacturing cost, demand function and return rate function;

(3) All returned products are suitable for remanufacturing and the remanufacturing costs are constant.

We note that several studies in the CLSC literature are based on similar assumptions (see e.g., Savaskan et al. 2004):

3.2 Problem Formulation

The profit of each member and the whole supply chain could found as follows. The retailer's profit $\prod^{R}$ is

$$\begin{aligned}\Pi^{R} &= D(p)(p-w)+D(p)r(c)(b-c)\\ &= D(p)[p-w+r(c)(b-c)]\end{aligned} \tag{1}$$

The retailer's profit contains two parts: one is the profit from the forward flow by selling every new product (*p*-*w*); the other is the profit from the reverse flow by collecting every used product and returning it to the manufacturer (*b*-*c*). In other words, the retailer's profit comes from acting firstly as the distribution agent and secondly as the collection agent. Recall that $p=\beta w$ and $c=\lambda b$.

The manufacturer's profit $\prod^{M}$ is

$$\begin{aligned}\Pi^{M} &= D(p)[w - cm(1 - r(c)) - cr\, r(c)] - D(p)r(c)b \\ &= D(p)[w - cm + r(c)(\Delta - b)]\end{aligned} \tag{2}$$

Due to remanufacturing, the average unit production cost of the manufacturer is $cm(1\text{-}r(c))+cr\ r(c)$, i.e., $1\text{-}r(c)$ proportion of total products will be manufactured from raw materials, and the remaining $r(c)$ proportion will be remanufactured from returned products. The manufacturer also has to pay the retailer for the collecting service. Similar to the retailer's profit, the manufacturer's profit can be interpreted as two parts: one is the profit from the forward flow by selling every new product (w-cm), the other is the cost savings from the reverse flow by remanufacturing every used product (Δ-b).

The total supply chain profit $\prod^{T}$ is the sum of the profit of the retailer and the manufacturer:

$$\Pi^{T} = \Pi^{R} + \Pi^{M} = D(p)[p - cm + r(c)(\Delta - c)] \tag{3}$$

The Stackelberg game of the retailer-driven supply chain is the following two-step optimization problem: First find the optimal response function of the manufacturer $b^{*}(\beta, \lambda)$ and $w^{*}(\beta, \lambda)$ given the retailer's declaration of β and λ.

$$\{b^{*}(\beta,\lambda), \mathrm{w}^{*}(\beta,\lambda)\} = \underset{b,w}{\arg \mathrm{Max}} \prod\nolimits^{\mathrm{M}}(\, p = \mathrm{w}\beta, \mathrm{c} = \lambda \mathrm{b}) \tag{4}$$

Then find the optimal solution for the retailer considering the manufacture's reaction function.

$$\{\beta^{*}, \lambda^{*}\} = \underset{\beta,\lambda}{\arg \mathrm{Max}} \prod\nolimits^{\mathrm{R}}(\, b = b^{*}(\beta,\lambda), w = \mathrm{w}^{*}(\beta,\lambda)) \tag{5}$$

The sufficient condition for the existence and uniqueness of a Stackelberg equilibrium of the retailer-driven supply chain is the concavity of Π^{M} with respect to b

and w and the concavity of the Π^R with respect to β and λ (Cachon and Netessine, 2006).

3.3 Linear Demand and Linear Return Rate

The analysis of the retailer-driven model under the general function form of $D(p)$ and $r(c)$ seems to yield few managerial insights and is often intractable. Instead, we utilize linear demand and linear return rate functions so as to obtain interesting managerial insights and economic implication. We do not consider nonlinear demand function for the retailer-driven model.

The Linear demand function is widely used in economics and engineering literature and assumed to be in the form $D(p)=u-vp$ ($u>0$, $v>0$, $u>vp$, see, e.g., Varian 1992).

The linear return rate function $r(c)$ in the linear model is assumed to be in the form $r(c)=kc$, where $c \in [0,1/k]$, in which k is the marginal return rate, i.e., one unit increase in c results in k unit increase of the product return rate. k reflects the customers' willingness to return products. $r(c)$ is bounded above by 1, when all the products are returned to the supply chain. $1/k$ is the upper bound of the collecting price, because any collecting price larger than $1/k$ would not improve the return rate any more, i.e., the feasible region of c is [0, 1/k]. The linear function of $r(c)$ in the model is a first-degree approximation of many actual return rate functions (See Figure 2).

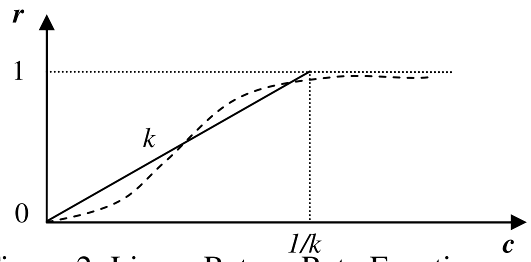


Figure 2. Linear Return Rate Function

For most cases with practical parameter values, we could assume that k is sufficiently small so that $0 \le c < 1/k$ (or $r(c)<1$) is always true for any reasonable c. In this paper, we will focus our attention on the case of $0 \le c < 1/k$ (i.e., $r(c)<1$), but we also give the solution for the case $c = 1/k$ (i.e., $r(c)=1$).

3.4 Intermediate Solution

In the retailer-driven model with linear demand and linear return rate functions, we obtain an intermediate solution as follows.

The manufacturer's profit given the retailer's announcement of β and λ is

$$\Pi^{M} = D(p)[w - cm + r(c)(\Delta - b)] = D(\mathrm{w}\beta)[w - cm + r(\lambda \mathrm{b})(\Delta - b)]$$

where $D(p)=u-vp$ and $r(c) = kc,\ c \in [0, 1/k]$.

We focus our attention on the case of $r(c)<1$. First we can find that the manufacturer's reaction functions by $Max\ \Pi^{M}$ over w and b. The optimal wholesale price w and optimal return price b given the retailer's announcement of β and λ are:

$$w^{*}(\beta, \lambda) = u/(2v\beta) + cm/2 - k\lambda\Delta^{2}/8 \quad \text{and} \quad b^{*}(\beta, \lambda) = \Delta/2$$

Notice $b^{*}(\beta, \lambda)$ is independent on β and λ, i.e., the optimal return price $b^{*} \equiv \Delta/2$.

Then consider $\Pi^{R} = D(p)[p-w+r(c)(b-c)]=D(w\beta)[w\beta-w+kb\lambda(b-b\lambda)]$. By taking into account the manufacturer's reaction functions $w^*(\beta,\lambda)$ and $b^*(\beta,\lambda)$, we find the retailer's problem is

$$Max\,\Pi^R(\beta,\lambda)=\frac{[4(\beta-1)(u+cm\,v\beta)-kv\beta\lambda\Delta^2(\beta+2\lambda-3)][4u+v\beta(k\lambda\Delta^2-4cm)]}{64v\beta} \quad (6)$$

After solving Expression (6) and finding the optimal β^*, λ^*, we can then find the optimal $w^*=w^*(\beta,\lambda)$, $b^*=\Delta/2$, $p^*=\beta^* w^*$, $c^*=\lambda^* b^*$, and the profit of each party and the total supply chain. The intermediate solution for the case of *r(c)=1* is given in Appendix I.

Expression (6) is a bivariate optimization problem, we could find the optimal solution of Expression (6) by first solving the first order necessary condition (FONC), $\{\partial\Pi^R/\partial\beta=0, \partial\Pi^R/\partial\lambda=0\}$ (see Appendix II for the difficulty of obtaining a closed-form solution from FONC). We can check if the FONC satisfying point is optimal by checking the second order sufficient condition (SOSC) evaluated at such a point, that is, checking the Hessian Matrix of $\Pi^R(\beta,\lambda)$, $H=\begin{pmatrix} h11 & h12 \\ h21 & h22 \end{pmatrix}=$

$$\begin{pmatrix} -\frac{16u^2+v^2\beta^3(-4cm+k\Delta^2\lambda)^2}{32v\beta^3} & \frac{1}{32}kv\Delta^2(k\Delta^2(3-2\beta-3\lambda)\lambda+8cm(-1+\beta+\lambda)) \\ -\frac{1}{32}kv\Delta^2(-8cm(-1+\beta+\lambda)+k\Delta^2\lambda(-3+2\beta+3\lambda)) & -\frac{1}{32}k\Delta^2(8u+v\beta(-8cm+k\Delta^2(-3+\beta+6\lambda))) \end{pmatrix}$$

To meet the SOSC, we should have

$$h11=-\frac{16u^2+v^2\beta^3(-4cm+k\Delta^2\lambda)^2}{32v\beta^3}<0 \quad (C1)$$

It can be easily verified that Condition (C1) holds.

Also, to meet SOSC, we should have *h11*h22-h12*h21*>0, i.e.,

$$h11*h22-h12*h21=\frac{1}{1024\,v\,\beta^3}\Big(k\,A^2\big(128\,u^3+8\,u\,v^2\,\beta^3\,(-4\,cm+k\,A^2\,\lambda)^2+16\,u^2\,v\,\beta\,(-8\,cm+k\,A^2\,(-3+\beta+6\,\lambda))+v^3\,\beta^3\,(-128\,cm^3\,\beta-3\,k^3\,A^6\,\lambda^2\,(\beta^2+3\,(-1+\lambda)^2+\beta\,(-3+2\,\lambda))+16\,cm^2\,k\,A^2\,(-3\,\beta^2-4\,(-1+\lambda)^2+\beta\,(5+2\,\lambda))+8\,cm\,k^2\,A^4\,\lambda\,(3\,\beta^2+6\,(-1+\lambda)^2+\beta\,(-7+3\,\lambda)))\big)\Big)>0 \quad \text{(C2)}$$

A sufficient condition to guarantee that there exists a unique optimal solution to the *Max* $\prod^{R}(\beta,\lambda)$ problem of Expression (6) is that Condition (C2) is met in the region of interest defined as $\beta \in [\beta_L, \beta_U]$ and $\lambda \in [\lambda_L, \lambda_U]$, and hence $\prod^{R}(\beta,\lambda)$ is concave in the region. $[\beta_L, \beta_U]$ and $[\lambda_L, \lambda_U]$ are meaningful and practical ranges of β and λ, and could be obtained from real economic data.

CHAPTER 4. NUMERICAL ANALYSIS

Even when Condition (C2) in Chapter 3 does not hold, we observe that a unique equilibrium may exist in numerous cases. For such cases, in this chapter, we present the numerical analysis of the retailer-driven model with linear function forms.

Section 4.1 gives a numerical example with a set of parameter values. Based on the numerical example, we then discuss the optimal λ in the supply chain, the meaning, and the relationship to the loss leader pricing policy. Section 4.2 presents extensive numerical examples and sensitivity analysis.

4.1 A Numerical Example and Optimal λ

We first consider a numerical example with u=1000, v=20, cm=5, k=0.5 and Δ=1 to introduce the findings of the model. The graph of the retailer's profit vs. β and λ (Expression (6)) is shown in the Figure 3.

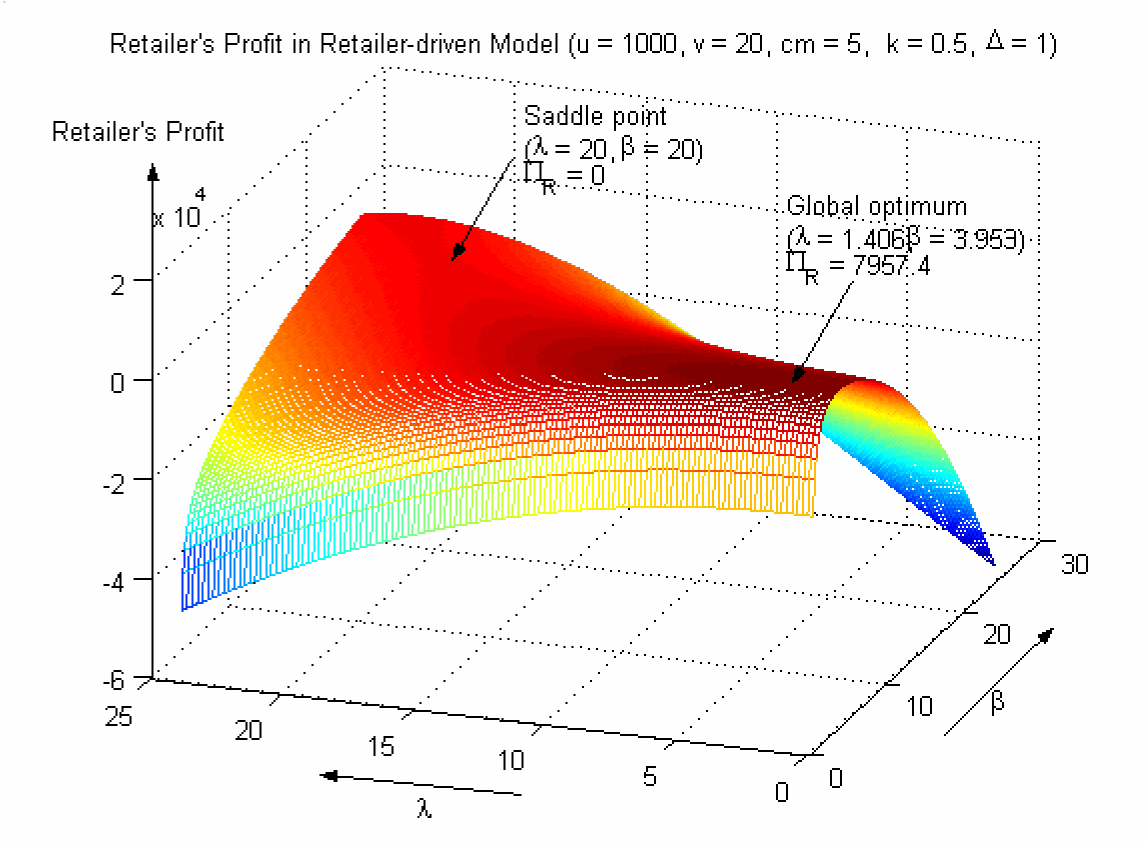


Figure 3. Retailer's Profit in Retailer-driven Model

In Figure 3, the retailer's profit function has one global optimal point at (β=3.953, λ=1.406, $\Pi^R = 7957.4$), and one saddle point at (β=20, λ=20, $\Pi^R = 0$). $\Pi^R(\beta, \lambda)$ is concave around the global point (β=3.953,λ=1.406).

We note that in the retailer-driven model with forward flow only (see Appendix XI) with the same parameter values, we find the optimal β=3.858 and the retailer's profit is Π^R=7881.71. The retailer-driven remanufacturing model gives more profit for the retailer and larger forward flow price multiplier.

One interesting point from the numerical example is that $\lambda^*>1$, which is quite counter-intuitive. To understand the meaning of $\lambda^*>1$, we need to consider that to achieve her profit maximization, the retailer has three possibilities:

(i) Profit from forward flow only, and may lose profit in reverse flow (β>1, λ≥1).

(ii) Profit from reverse flow only, and may lose profit in forward flow (β≤1, λ<1).

(iii) Profit from both forward flow and reverse flow (β>1, λ<1).

The numerical example in Figure 3 shows that the retailer not only gives up the profit from the reverse flow, but also incurs loss in the reverse flow to achieve the maximal profit from the forward flow by setting β>1, λ>1 (when the return rate *r(c)* is less than 1).

λ^*>1 seems to imply that by incurring loss in the reverse flow, the retailer helps the manufacturer to collect more used product and lower the unit production cost ($c_m(1-r(c))+c_r\, r(c)$). The lowered unit production cost enables the manufacturer charge less wholesale price *w*, which compensates the retailer's loss in the reverse flow. In this way, the retailer's profit can be maximized.

This phenomenon is very similar to the loss leader pricing policy used by many retailer stores to attract customers by setting very low prices, sometimes below cost, for some products and hoping the customers will buy other products to make up the loss and also make enough profit. We define "strong loss leader pricing" as the pricing strategy in which the undertaker incurs strict loss for the product, and "weak loss leader pricing" as the pricing strategy in which no profit is made or loss is incurred. In this example of the retailer-driven model, we observe that the retailer, the Stackelberg leader in the supply chain, adopts the strong loss leader pricing policy to the reverse flow.

A question about this loss leader pricing phenomenon is how often we could expect it. We conduct extensive numerical examples in section 4.2 to show that λ>1 is prevailing.

4.2 Numerical Examples and Sensitivity Analysis

In this section, we conduct extensive numerical examples to show how prevailing the loss leader pricing phenomenon could be observed with various parameter values.

The numerical example in section 4.1 uses the following parameter values u=1000, v=20, cm=5, k=0.5 and Δ=1. Among all the parameters, we believe that the direct savings from remanufacturing (Δ) and the marginal return rate (k) are the most influential in the reverse flow, because Δ reflects the potential savings of the reverse flow for the supply chain and k reflects the customers' willingness to return products. The slope of the demand function (v) is also important because it affects the demand in the forward flow.

In our numerical examples, first we study the effect of the different combination of k values (from k=0.05 to 1.6) and Δ values (Δ=1,2,3) on the retailer-driven supply chain, especially on λ. The details of the numerical examples could be found in Appendix V.

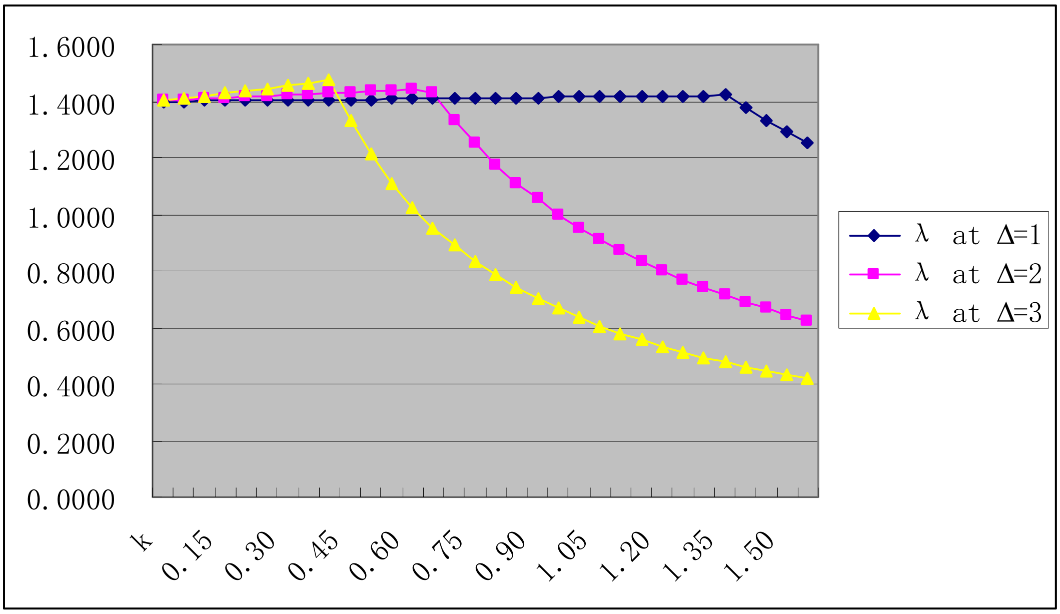


Figure 4. Optimal λ vs. k and Δ in Retailer-driven Model

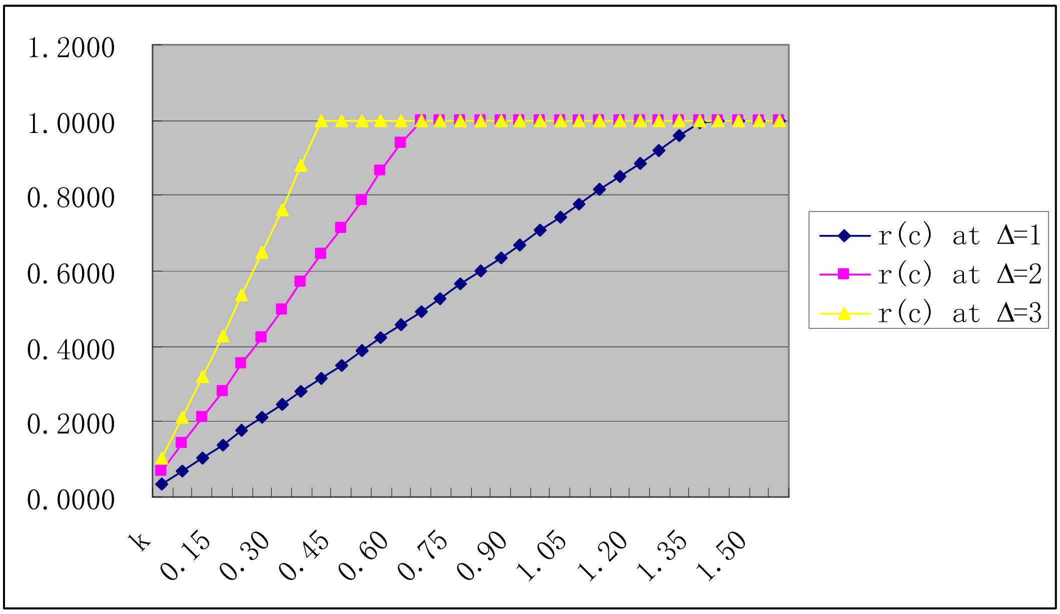


Figure 5. Return Rate *r(c)* vs. *k* and *Δ* in Retailer-driven Model

We first look at the reverse flow price multiplier λ (see Figure 4 and Figure 5). We observe that each curve in Figure 4 first increases slowly and then decreases sharply in *k*. The turning point of the two segments of each curve is when the return rate $r(c)$ hits 1, i.e., when $r(c) <1$, λ is increasing in k and when $r(c)=1$, λ is decreasing in k.

k represents the customers' willingness to return products. As *k* increases, the retailer reacts to collecting more used products by setting a slightly higher λ and $\lambda >1$ (this is the loss leader pricing policy); the process continues until the return rate hits 1, and then the retailer starts to decrease λ because a high λ is not necessary when the return rate hits 1. The optimal λ when return rate $r(c)=1$ is $\lambda^*=2/(\Delta k)$ and follows a different pricing mechanism from loss leader pricing (see Appendix I).

What we observe from the above is that the loss leader pricing phenomenon is prevailing in the retailer's pricing strategy with different *k* and *Δ* values for the cases of r(c)<1. The second interesting observation is that the optimal λ in the retailer-driven model is very insensitive to the k and Δ values, as we can see in Figure 4, λ^* is around 1.4 for the cases of r(c)<1, regardless of the fact that k changes from 0.05 to 1.6 and Δ changes from 1 to 3. (The discussion of the sensitivity of the forward price multiplier β with respect to k and Δ could be found in Appendix V).

We continue our numerical examples to study another important parameter v, the slope of the demand function, and its impact on the optimal λ of the retailer-driven model. We use the parameter values u=1000, cm=5, k=0.15 and Δ=2, and vary v from 5 to 199. Because $p \geq cm$, and for a positive $D(p)=u-vp$ exists, v has be $v \leq u/cm$=200. See Figure 6 for the optimal λ vs. v in the retailer-driven model for this example (see Appendix V for detailed data of the numerical examples).

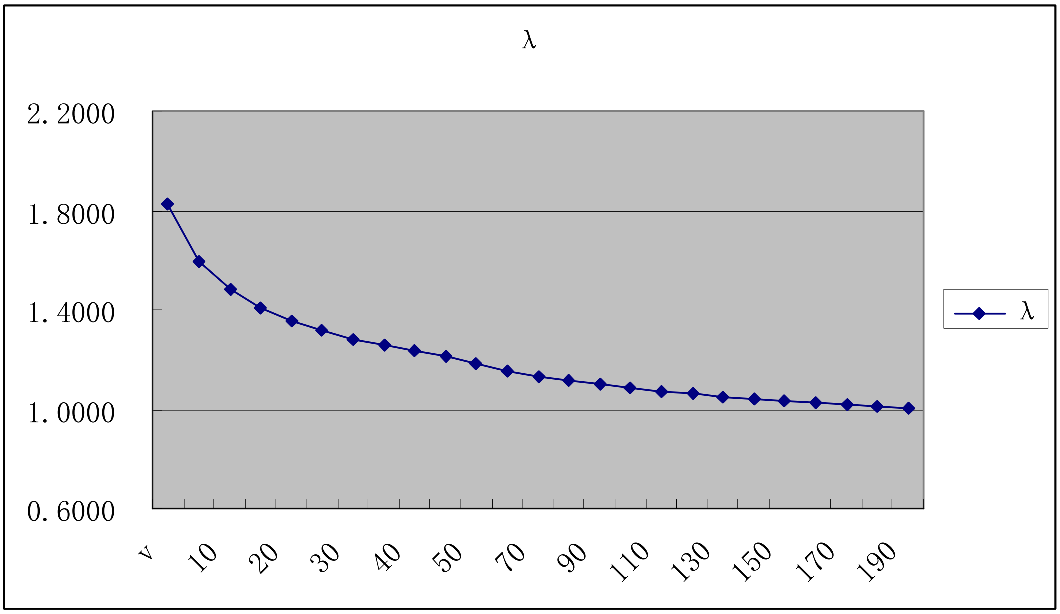


Figure 6. Optimal λ vs. v in Retailer-driven Model

From Figure 6, we observe that (1) the λ is always greater than 1 in all the feasible range of v, which confirms that the loss leader pricing is prevailing; and (2) the optimal λ

is more sensitive to v compared to k or Δ. The observation seems to suggest that the forward flow is more important to the retailer in terms of profitability and thus the retailer would adopt a loss leader pricing policy to the reverse flow.

Based on the numerical examples, we summarize the key findings of the retailer-driven model in the following observation.

OBSERVATION 1. ***The retailer (Stackelberg leader) in the two-echelon retailer-driven closed-loop supply chain with product remanufacturing gives up the profit from reverse flow and makes profit from the forward flow only, by setting the optimal price multiple*** $\lambda^*>1$***. The optimal*** λ^* ***is sensitive to the slope of the demand function (v), but insensitive to the direct savings from remanufacturing (Δ) or the marginal return rate (k).***

CHAPTER 5. OPTIMALITY ANALYSIS

In this chapter, we provide an observation on λ via a limited optimality analysis of the retailer-driven model at λ=1. That is, for this chapter, we do not assume the concavity of the retailer's objective function everywhere. The purpose of the chapter is to help gain an intuitive explanation of the optimal λ^* being $\lambda^*>1$ with the help of a Retailer-driven Manufacturer-collecting Model.

5.1 Optimality Analysis at λ=1

The basic retailer-driven model has a special case when $\lambda=1$ (as shown in the upper section of Figure 7), which is mathematically equivalent to the Retailer-driven Manufacturer-collecting Model (as shown in the lower section of Figure 7).

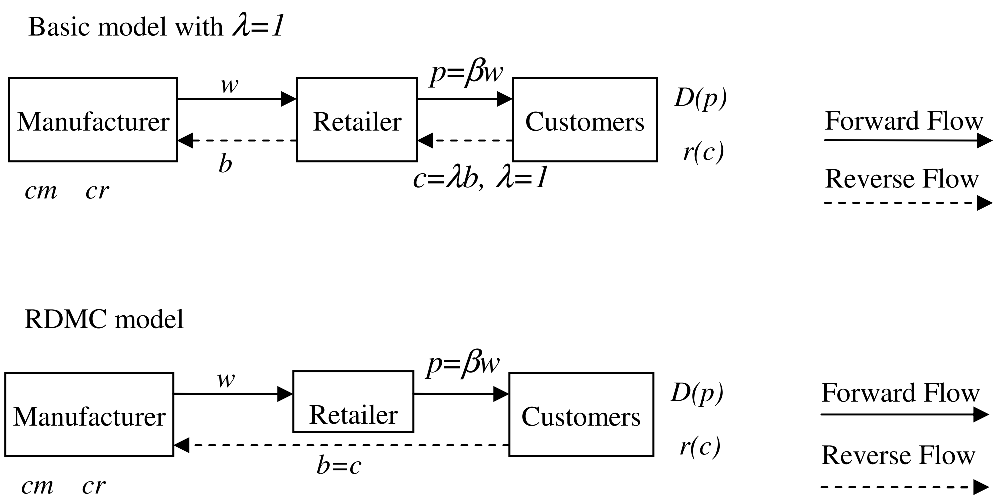


Figure 7. Retailer-driven Manufacturer-collecting (RDMC) Model

In the Retailer-driven Manufacturer-collecting (RDMC) Model, the manufacturer collects the used product directly from the customers; and the retailer only operates in the forward flow, by declaring the forward flow price multiplier β. We use the notation $\Pi^{R}_{RDMC}(\beta)$ to represent the profit of the retailer as a function of the sole decision variable β in the RDMC model. The retailer is still the Stackelberg leader and all other assumptions are still true in the RDMC model. This situation is equivalent in terms of prices and profits to our basic retailer-driven (retailer-collecting) model with $\lambda=1$.

Assume that ($\beta=\beta^2>1$) is the optimal solution of the RDMC model, obviously this solution is equivalent to the point ($\beta=\beta^2,\lambda=1$) in the basic retailer-driven (retailer-collecting) model, as shown in the Figure 8 point B.

We also assume that $4cm - k\Delta^2 > 0$ (which is a reasonable assumption for practical parameter values because k is usually a small number) and Let $\beta^1=\frac{2\sqrt{u}}{\sqrt{v(4cm - k\Delta^2)}}$, we could show that $\left.\frac{\partial \Pi^R(\beta,\lambda)}{\partial \lambda}\right|_{(\beta=\beta^1,\lambda=1)}=0$ and $\left.\frac{\partial \Pi^R(\beta,\lambda)}{\partial \lambda}\right|_{(\beta>\beta^1,\lambda=1)}>0$ (see appendix III for proof). This means that if we use gradient-based optimization at any point ($\beta>\beta^1$, $\lambda=1$) as shown in the bold dashed line in Figure 8, the next step is pointing to somewhere $\lambda>1$.

Moreover, we could show that $\beta^2>\beta^1$ (see appendix IV for proof) when $4cm - k\Delta^2 > 0$ and $u > v(4cm - k\Delta^2)$ (both inequalities are reasonable assumptions for practical parameter values because k is usually a small number, and u is usually a big number.) This in turn means $\left.\frac{\partial \Pi^R(\beta,\lambda)}{\partial \lambda}\right|_{(\beta=\beta^2,\lambda=1)}>0$.

We also know that $\left.\frac{\partial \Pi^R(\beta,\lambda)}{\partial \beta}\right|_{(\beta=\beta^2,\lambda=1)}=0$ because ($\beta=\beta^2,\lambda=1$) is the optimal solution of the RDMC model.

Considering the above, we conclude that the gradient of the function $\Pi^R(\beta,\lambda)$ at the point ($\beta=\beta^2,\lambda=1$) is $\nabla_{(\beta,\lambda)}\Pi^R(\beta,\lambda) = (0, \left.\frac{\partial\Pi^R(\beta,\lambda)}{\partial\lambda}\right|_{(\beta=\beta^2,\lambda=1)} > 0)$. Figure 8 shows the gradients, β^1, β^2 and the actual global optimum of the numerical example used in section 4.1.

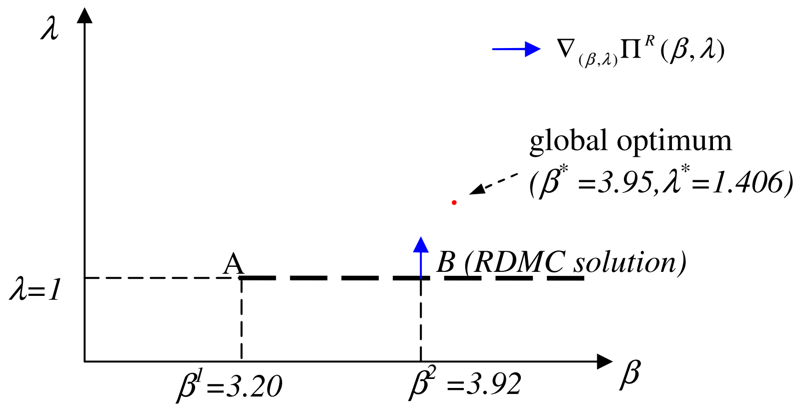


Figure 8. Optimality Analysis at $\lambda=1$ (u=1000, v=20, cm=5, k=0.5 and Δ=1)

The above analysis is interesting, because from the optimization point of view, if using some gradient-based search method and starting at point B (the optimal solution of RDMC model), even though we don't know where the search converges, the initial move is to somewhere ($\beta=\beta^2,\lambda>1$).

Intuitively, we could image that when the supply chain moving from the RDMC model to our basic retailer-driven (retailer-collecting) model, as the retailer gains more control (mathematically from fixed $\lambda=1$ to a unconstrained λ), the initial move of the retailer is to keep the forward flow pricing strategy β intact (because

$\left.\frac{\partial \Pi^{R}(\beta,\lambda)}{\partial \beta}\right|_{(\beta=\beta^{2},\lambda=1)}=0$); while at the same time to set the reverse flow pricing strategy as $\lambda > 1$, which is exactly the loss leader pricing phenomenon we observe, i.e., the retailer adopts loss leader pricing policy to the reverse flow.

CHAPTER 6. CENTRALLY COORDINATED AND MANUFACTURER-DRIVEN MODELS

The centrally coordinated and manufacturer-driven supply chain with product remanufacturing has been studied in a few literatures. Among other, Savaskan (2004) studies such models with the assumption that the total cost of collecting the used product is $C(\tau) = I + A\tau D(p) = C_L\tau^2 + A\tau D(p)$, where *I* is the marketing investment, τ is the return rate, *A* is the constant variable unit cost paid to the customers for returning a used product, C_L is a scaling parameter, and *D(p)* is the demand. In their model, the return rate is a result of the marketing investment *I*, i.e., $\tau = \sqrt{\mathrm{I/C}}$ and the variable unit cost (*A*) paid for returning a used product is not a decision variable.

We take a different approach to model the centrally coordinated and manufacturer-driven models with the assumption that the return rate, *r(c)*, is a direct result of the incentive paid (*c*).

In this chapter, section 6.1 presents the model description of the centrally coordinated and the manufacturer-driven model. We then analyze the centrally coordinated model in section 6.2 and the manufacturer-driven model in section 6.3, which is followed by the optimality analysis of the solution of the manufacturer-driven model in Section 6.4. Section 6.5 gives the result of the linear models.

6.1 Model Description

We only give a brief description of the centrally coordinated and manufacturer-driven model below, because the two models have been studied in a few literature (even through with different assumption).

See Figure 9 for the three models: centrally coordinated model, manufacturer-driven model and retailer-driven model (to provide a comparison here). In this paper, we also refer the three models as three scenarios: Scenario C, Scenario M, and Scenario R to make reference easy.

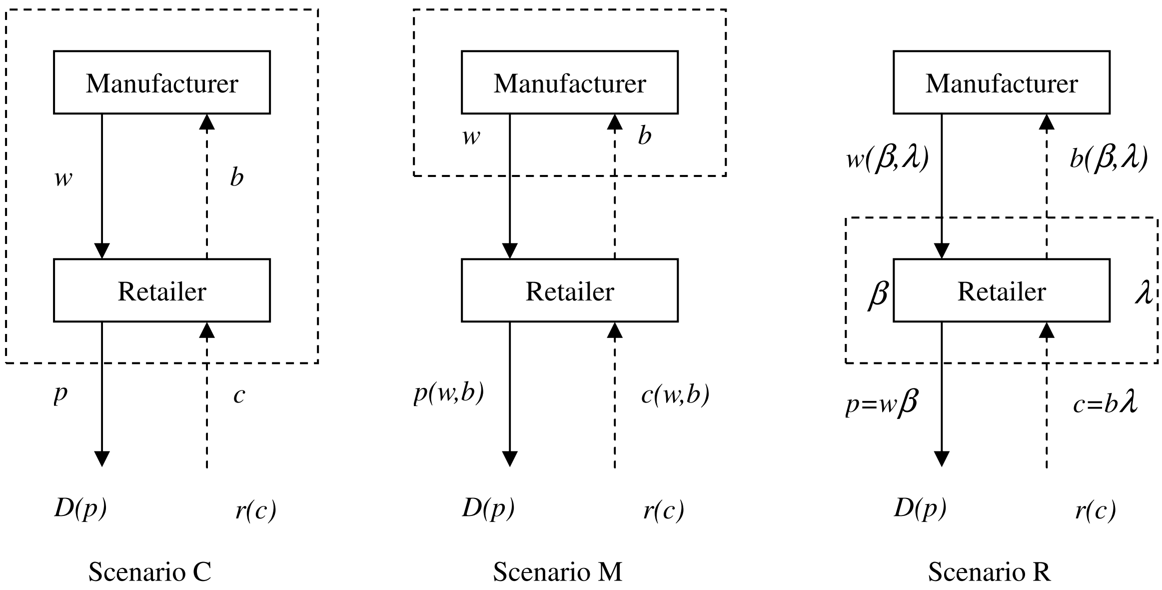


Figure 9. Three Scenarios of the Supply Chain Power

In Scenario C, the manufacturer and the retailer, considered as a whole system, jointly determine the optimal prices *p* and *c* to maximize the total supply chain profit. The prices *w* and *b* do not affect the total supply chain profit because they are within the system; but they affect how the total profit is divided between the manufacturer and the retailer. In the later of the paper, we also discuss some coordination mechanisms to implement the centrally coordinated supply chain. Scenario C serves as the benchmark in our research to highlight the inefficiency of the other two scenarios.

Scenario M and Scenario R represent two cases where dominant supply chain power exists in the supply chain. We assume that no coordination exists in Scenario M and Scenario R.

In Scenario M, the manufacturer has sufficient supply chain power over the retailer to act as a Stackelberg leader. The manufacturer can predict the retailer's reaction (setting of prices p and c) given his action (setting of w and b). By taking into account the retailer's reaction the manufacturer tries to obtain his maximal profit.

We have discussed Scenario R (retailer-driven channel) in details in Chapter 3 and 4.

We note that in Scenario R, the price multiplier β has a special meaning in the economics literature, i.e., the sensitivity of the retail price to the wholesale price ($\beta=\partial p/\partial w$). For example, in the classic bilateral monopoly model, the sensitivity β is always 1/2 (see Tirole, 1988; Varian, 1992). Accordingly, the price multiplier λ ($\lambda=\partial c/\partial b$) can be interpreted as the sensitivity of the collecting price to the return price. While in Scenario M the Stackelberg leader (the manufacturer) first directly determining the prices and thus indirectly determines the sensitivities; in Scenario R the Stackelberg (the retailer) directly determines the sensitivities, which in turn indirectly determine the prices. In this way, Scenario M and Scenario R are unified in the same framework of optimal pricing decision.

In this chapter, we use general function form of $D(p)$ and $r(c)$ for analysis. We assume that $D(p)>0$, $D'(p)<0$ and $D''(p)$ exists; $r(0)=0$, $0 \le r(c) \le 1$, $r'(c)>0$ and $r''(c)$ exists. For numerical examples, we use linear $D(p)$ and $r(c)$. We also note that conventional FONC's and SOSC's can be easily derived as in Chapter 3.

In the rest of the dissertation, we will use subscripts C, M, R of a variable to represent the three scenarios, e.g., b_M is the return price in Scenario M.

6.2 Centrally Coordinated Supply Chain

In centrally coordinated supply chain (Scenario C), the manufacturer and the retailer jointly maximize the total supply chain profit $\Pi^T = D(p)[p - cm + r(c)(\Delta - c)]$ over *p* and *c*. We can find that optimal collecting price c_C^* is independent on *p*, because for any given *p* the c_C^* is always

$$c_C^* = \arg\underset{c}{\text{Max}}\ r(c)(\Delta - c) \tag{4}$$

The first order necessary condition (FONC) and the second order sufficient condition (SOSC) at c_C^* are $r'(c)(\Delta\text{-}c)\text{-}r(c)=0$ and $r''(c)(\Delta\text{-}c)\text{-}2\,r'(c)<0$ respectively. We assume that SOSC is satisfied and the single global optimum is obtained at c_C^*. Once obtaining the optimal c_C^*, the optimal retail price p_C^* can be obtained in a similar way, i.e., by optimizing a univariate problem over *p*.

$$p_C^* = \arg\underset{p}{\text{Max}}\ D(p)[p - cm + r(c_C^*)(\Delta - c_C^*)] \tag{5}$$

We also assume that the SOSC at p_C^*, $2D'(p) + [r(c_C^*)(\Delta - c_C^*) - cm + p]D''(p) < 0$ is satisfied and the single global optimum is obtained at p_C^*.

As a result of the above assumptions and the specific structure of the objective function, we can conclude that the supply chain profit will be maximized at the single point (p= p_C^*, c=c_C^*).

6.3 Manufacturer-driven Supply Chain

In the manufacturer-driven supply chain (Scenario M), the manufacturer acts as the Stackelberg leader. The retailer maximizes $\prod^{R}$ over p and c for a given pair of w and b. Because the manufacturer knows exactly the retailer's reaction functions, i.e., optimal retail price $p^{*}(w,b)$ and optimal collecting price $c^{*}(w,b)$; By taking into account these reaction functions, the manufacturer maximizes $\prod^{M}$ over w and b. We present the key findings of Scenario M under general function form as follows.

PROPOSITION 1. ***In Scenario M (Manufacturer-driven model), to achieve his profit maximization, the manufacturer sets the optimal return price*** $b_M^{*}=\Delta=cm-cr$***, i.e., he profits from the forward flow only by adopting weak loss leader pricing policy to the reverse flow.***

Proof. See Appendix VI for a proof using implicit function theory. ■

Similar to the retailer in the retailer-driven model, the manufacturer in the manufacturer-driven model has three possibilities to achieve his profit maximization:

(i) Profit from forward flow only ($w>cm$, $b\geq\Delta$).

(ii) Profit from reverse flow only ($w\leq cm$, $b<\Delta$).

(iii) Profit from both forward flow and reverse flow ($w>cm$, $b<\Delta$).

Proposition 1 shows that when the manufacturer has dominant supply chain power, he should adopt the weak loss leader pricing policy to the reverse, similar to what the retailer does in the retailer-driven model, i.e, by giving up the cost savings from the reverse flow (Δ) to obtain the maximal profit from the forward flow only.

This is interesting because we observe that in both retailer-driven and manufacturer-driven model, the Stackelberg leader adopts the loss leader pricing policy to the reverse flow, which apparently highlights the relative importance in terms of profitability of the forward flow and the reverse flow.

We note that Savaskan et al. (2004) has the same observation (i.e., $b_M^* = \Delta$) under the assumption of a linear demand function and a specific collecting function, i.e., $C(\tau) = I + A\tau D(p) = C_L \tau^2 + A\tau D(p)$. Proposition 1 gives the conclusion in a more general form.

There seems to be a driving force behind this phenomenon. The manufacturer gives up all the savings from the reverse flow by setting $b=\Delta$. On the other hand, by setting $b=\Delta$, the retailer is given more incentive to collect the used product. Since the total returned product is $D(p)r(c)$, the retailer can decrease retail price p ($D'(p)<0$) or increase collecting price c ($r'(c)>0$) or do both to collect more. It appears that by setting $b=\Delta$, the manufacturer triggers the decrease in p (and thus the increase in demand $D(p)$), which in turn compensates the manufacturer's loss (or having no profit) in the reverse flow and it proves to be the optimal solution for the manufacturer in Scenario M. We illustrate this process by graphs using the linear demand function and linear return rate function in the next section 6.4.

As a comparison to the optimal $b_M^* = \Delta$ in the manufacturer-driven model, an interesting finding about the optimal b_R^* in the retailer-driven model is that the manufacturer sets the optimal return price $b_R^* < \Delta = cm - cr$, i.e, he should retain part of the direct savings from remanufacturing (see Appendix VII for proof). The implication is that when the manufacturer loses the leader position in the supply chain, he should not give up profit from the reverse flow. The reason is as follows. In Scenario M, considering the retailer's reaction functions, the manufacturer's profit is $\Pi_M^M = D(p(w,b))[w - cm + r(c(b))(\Delta - b)]$, where the retail price p is a function of w and b, and the collecting price c is a function of b (See Appendix VI). By observing Π_M^M, we can see that the demand could be changed by changing b, and that is the basis for the manufacturer's deliberate forfeit of profit in the reverse flow. However in Scenario R, given the retailer's announcement of β and λ, the manufacturer's profit is $\Pi_R^M = D(\beta w)[w - cm + r(b\lambda)(\Delta - b)]$. By observing Π_R^M, we can see b has no effect on the demand by the customers, which means that there is no reason to give up the profit in the reverse flow. Therefore, the supply chain power structure determines the manufacturer's decision of the return price.

The discussion regarding other optimal prices (p^*,w^*,b^*) and profits in Scenario M could be found in Appendix VIII.

6.4 Optimality Analysis of b_M^* =Δ in Scenario M with the linear function forms

This section shows the features around the optimal point b_M^* =Δ in Scenario M with the linear function forms, i.e., the features of the model when b=$b_M^* \pm \varepsilon$ (ε is a small positive real number). This section partially explains the underling reason why the manufacturer gives up all the directing savings Δ from the reverse flow (for the case $r(c)<1$).

The optimal solution of the manufacturer's profit is (b_M^*, w_M^*), but our focus is only the return price b and our approach is to study the path from $(b, w^*(b))$ to (b_M^*, w_M^*), where $b = b_M^* \pm \varepsilon$ and $w^*(b)$ is optimal wholesale price for the manufacturer given the return price b.

Following the procedure described in Appendix VI and assuming the case $r(c)<1$, we first obtain the retailer's reaction functions $p^*(w,b)$ and $c^*(b)$, then replace these reaction functions into the manufacturer's profit Π^M. We can get Π^M as a function of w and b, i.e.,

$$\Pi^M(w,b) = (2w - kb(b+cr) + cm(kb-2))(4u + kvb^2 - 4vw)/16$$

To obtain $w^*(b)$, we solve $\partial\Pi^M(w,b)/\partial w = 0$ and get

$$w^*(b) = u/(2v) + cm/2 + (3kb^2 - 2kb\Delta)/8$$

We can then find the manufacturer's profit as a function of b:

$$\Pi^M(b) = \Pi^M(w^*(b),b) = (4u - v(4cm + kb(b-2\Delta)))^2/(256v)$$

and the retail price as a function of b:

$$p^*(b) = p^*(w^*(b), b) = \Pi^M(b) = \Pi^M(w^*(b),b) = (12u + v(4cm + kb(b-2\Delta)))/(16v)$$

To further study the effect of b on the forward flow and the reverse flow, we calculated the manufacturer profit from the forward flow as

$$\Pi^{MF}(b) = (u - vp^*(b))(w^*(b) - cm) = (4u - v(4cm + kb(b-2\Delta)))(4u - v(4cm - kb(3b-2\Delta)))/(128v)$$

and the manufacturer's profit from the reverse flow as

$$\Pi^{MR}(b) = (u - vp^*(b))(kc(\Delta - b)) = (4u - v(4cm + kb(b-2\Delta)))kb(\Delta - b)/32$$

A graphic illustration of the relationships among $\Pi^M(b)$, $\Pi^{MF}(b)$, $\Pi^{MR}(b)$ and $p^*(b)$ around the optimal point $b_M^* = \Delta$ is shown in Figure 10. We can see that the total profit is maximized at $b=\Delta$, while the retail price is maximized at the same point $b=\Delta$ (which means the demand is maximized at b=Δ). The forward flow profit $\Pi^{MF}(b)$ is increasing and the reverse flow profit $\Pi^{MR}(b)$ is decreasing around b_M^* =Δ; but the abstract value of the slope of $\Pi^{MF}(b)$ is greater than that of $\Pi^{MR}(b)$ when b<Δ, and vice verse when b>Δ.

The relationships partially explain the underlying reason why the manufacturer gives up Δ from the reverse flow. When b increases to Δ from a point b<Δ, the retail price decreases and the demand increases. The increased demand benefits the manufacturer in the forward flow. Although the profit from the reverse flow decreases, the total profit increases. A similar analysis can be done when b>Δ.

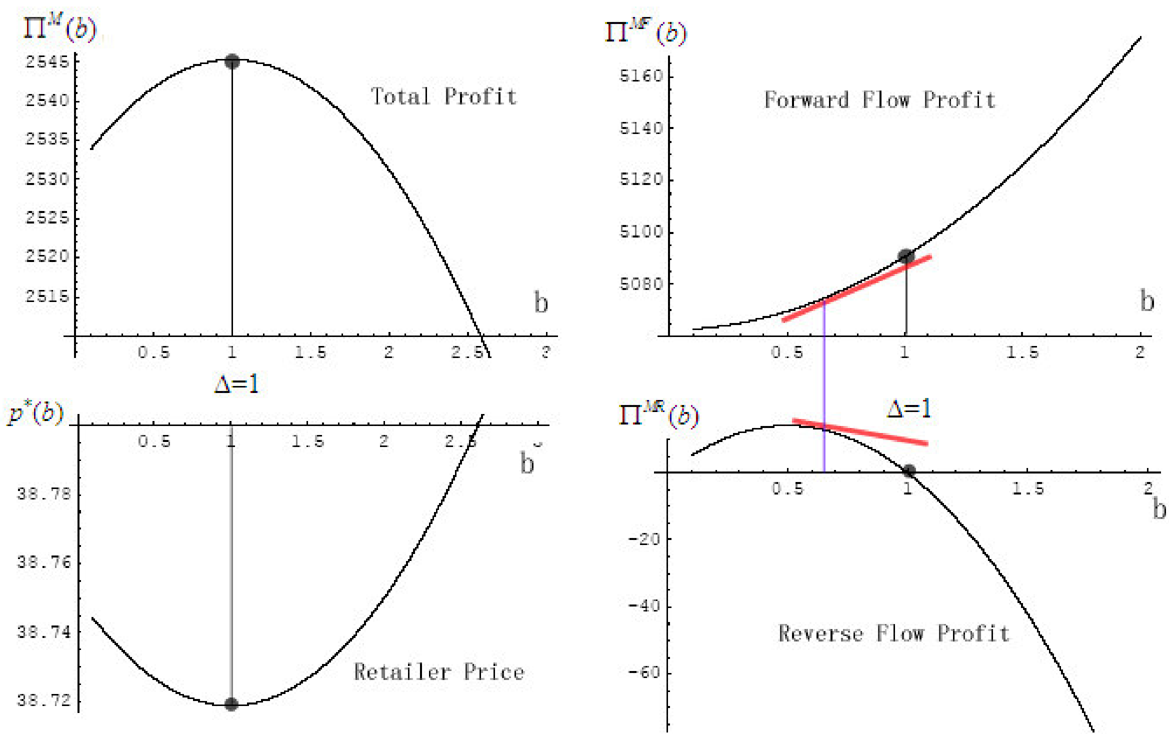


Figure 10. Optimality Analysis at b_M^* =Δ (*u*=1000, *v*=20, *cm*=5, Δ=1, *k*=0.5.)

6.5 Linear model - Scenario C and Scenario M with the linear function forms

In this section, we briefly give the result of the solution of Scenario C and Scenario M with the linear demand and return rate function forms. Following a similar solution procedure (see Appendix VI), we can find the closed-form solutions of the prices and the profits in the supply chain for Scenario C and Scenario M with the linear function forms (see Table 2). To further compare our model with the traditional model with only forward flow, we also include in Table 2 the corresponding solutions of the traditional model.

Table 2. Linear Model Results (Scenario M and Scenario C)

	Scenario M (our model)		Scenario C (our model)		Scenario M (forward only model)	Scenario C (forward only model)
	If $\Delta/2<1/k$ (Unbound solutions)	If $\Delta/2\geq 1/k$ (Bounded solutions)	If $\Delta/2<1/k$ (Unbound solutions)	If $\Delta/2\geq 1/k$ (Bounded solutions)		
w^*	$\frac{u}{2v}+\frac{cm}{2}+\frac{k\Delta^2}{8}$	$\frac{u}{2v}+\frac{cr}{2}+\frac{3}{2k}+\varepsilon$ *	n/a	n/a	$\frac{u}{2v}+\frac{cm}{2}$	n/a
b^*	Δ	$2/k+\varepsilon$ *	n/a	n/a	n/a	n/a
p^*	$\frac{3u}{4v}+\frac{cm}{4}-\frac{k\Delta^2}{16}$	$\frac{3u}{4v}+\frac{1}{4k}+\frac{cr}{4}$	$\frac{u}{2v}+\frac{cm}{2}-\frac{k\Delta^2}{8}$	$\frac{u}{2v}+\frac{1}{2k}+\frac{cr}{2}$	$\frac{3u}{4v}+\frac{cm}{4}$	$\frac{u}{2v}+\frac{cm}{2}$
c^*	$\Delta/2$	$1/k$	$\Delta/2$	$1/k$	n/a	n/a
Π^M	$\frac{(u-(cm-k\Delta^2/4)v)^2}{8v}$	$\frac{(u-(cr+1/k)v)^2}{8v}$	n/a	n/a	$\frac{(u-cm\cdot v)^2}{8v}$	n/a
Π^R	$\frac{(u-(cm-k\Delta^2/4)v)^2}{16v}$	$\frac{(u-(cr+1/k)v)^2}{16v}$	n/a	n/a	$\frac{(u-cm\cdot v)^2}{16v}$	n/a
Π^T	$\frac{3(u-(cm-k\Delta^2/4)v)^2}{16v}$	$\frac{3(u-(cr+1/k)v)^2}{16v}$	$\frac{(u-(cm-k\Delta^2/4)v}{4v}$	$\frac{(u-(cr+1/k)v)^2}{4v}$	$\frac{3(u-cm\cdot v)^2}{16v}$	$\frac{(u-cm\cdot v)^2}{4v}$

* $\varepsilon\geq 0$.

Notice from Table 2 that depending on whether $\Delta/2 < 1/k$ or not, the optimal solution is drawn from one set of two solutions for Scenario M and Scenario C in our model (one is unbound by c≤1/k or $r(c) \leq 1$, the other is bounded by c≤1/k or $r(c) \leq 1$). We summarize some interesting findings of Scenario C and Scenario M of the linear model as follows.

[1] Our model is a generalization of the traditional forward-only model. Specifically when remanufacturing is impossible (k=0 or Δ=0), our model is degenerated into the traditional forward-only model.

[2] When $\Delta/2 < 1/k$, the optimal return rate is less than 1; when $\Delta/2 \geq 1/k$, the optimal return rate is 1.

[3] In Scenario M of our model, when the return rate is less than 1 ($\Delta/2 < 1/k$), the optimal return price is b_M^* =Δ (Proposition 1) and the optimal collecting price is c_M^* =Δ/2.

[4] When the optimal return rate is 1 (i.e., when $\Delta/2 \geq 1/k$), the manufacturer can retain part of the direct savings from remanufacturing in Scenario M, i.e., $b_M^* = 2/k + \varepsilon$ could be less than Δ (This is different from Proposition 1 and caused by the indifferentiability of $r(c)$ at the point c=1/k). The term ε in b_M^* and w_M^* in Scenario M means that when all the products will be returned (i.e., $r(c)$=1), only the difference between w_M^* and b_M^* affects the manufacturer's profit, not the exact values.

[5] $\Pi^M = 2\Pi^R = 2\Pi^T/3$ in Scenario M of both our model and traditional model.

[6] In both our model and the traditional model, the total supply chain profit Π^T of Scenario M shrinks 25% compared to that of Scenario C. Since both models show the

same percentage of shrinkage of the total profit, we can conclude that the shrinkage is caused entirely by the forward flow in Scenario M, i.e., the reverse flow in Scenario M causes no inefficiency.

CHAPTER 7. DISCUSSION OF THE MODELS

In this chapter, we present some discussion about the three models (retailer-driven, manufacturer-driven and centrally coordinated), which includes the comparison of the three models in section 7.1, the supply chain coordination issues in section 7.2, the policy implication of the models in section 7.3, and a third party collecting model in section 7.4.

7.1 Comparison of the Models

First we use a simple numerical example with parameter values (u=1000, v=20, cm=5, cr=4, k=0.5) to illustrate the comparison of the models with linear function forms. For Scenario C and Scenario M, we use Table 2 to compute the prices and the profit. For Scenario R, we can find the solution using the methods in Chapter 3.

For the three scenarios, we also calculate an index called "Efficiency" (Efficiency = Total supply chain profit of the scenario / Total supply chain profit of Scenario C) to reflect the total supply chain profit as a percentage of the maximal achievable profit of the supply chain. The results are in Table 3.

Table 3. Numerical Comparison of the Three Scenarios

	w^*	p^*	β^*	b^*	c^*	λ^*	c^*/p^*	Π^M	Π^R	Π^T	Efficiency*
C		27.44			0.5		0.018			10181.33	100%
M	27.56	38.72	1.41	1	0.5	0.5	0.012	5090.66	2545.33	7635.99	75%
R	8.74	34.55	3.95	0.5	0.70	1.41	0.020	1210.3	7957.4	9167.7	90.04%

*Efficiency = Total supply chain profit of the scenario / Total supply chain profit of Scenario C

From Table 3, we can notice that the supply chain power has great influence on the profit of each party in the supply chain. We observe that the majority of the profit is obtained by the Stackelberg leader; and that the leader in Scenario R (the retailer) extracts relatively more profit than the leader in Scenario M (the manufacturer).

Regarding the reverse flow in the supply chain, in Scenario M we observe that the manufacturer (the leader) gives up all the direct savings from remanufacturing in the reverse flow (by setting $b_M^* = cm - cr = 1$ as stated in Proposition 1) and profits from the forward flow only. In Scenario R, we observe that the retailer (the leader) not only gives up the profit from the reverse flow, but also actually incurs net loss from the reverse flow by setting $\lambda_R^* = 1.406 > 1$, i.e., by setting the collecting price c_R^* greater than the return price b_R^* (as stated in Observation 1). This is the loss leader pricing policy used by the Stackelberg leader in the supply chain.

Another interesting observation from the numerical example is that the overall supply chain of Scenario R is much more efficient than that of Scenario M (90.04% Efficiency of Scenario R vs. 75% Efficiency of Scenario M; also notice that the Efficiency of Scenario M is always 75%, see Table 2). The underlying reason appears to be the retailer's closeness or direct influence over the customers (i.e., the decision of the retail price p and the collecting price c has direct influence over the demand and return rate). When the retailer has supply chain power and the advantage of direct influence over market, the system efficiency is much higher than the situation when the manufacturer has supply chain power and the disadvantage of indirect influence over the market. This advantage of the retailer because of being closer to the customers has been reported for a different setting in a few literatures (e.g., Savaskan 2004 shows that retailer is the most effective undertaker of product collection activity because of being closer to the customers).

We then compare the supply chain performance in Scenario M and Scenario R. Regarding the supply chain efficiency in Scenario M, we know from Table 2 that Efficiency=$\Pi_M^T / \Pi_C^T = 75\%$; In Scenario R, the efficiency in the numerical examples with different parameter values is listed in Table 4. We observe that the total supply chain of Scenario R is much more efficient than that of Scenario M (average 90.6% Efficiency of Scenario R vs. 75% Efficiency of Scenario M). We also observe that the efficiency of Scenario R is increasing in *k* and in Δ), i.e., as the savings from remanufacturing Δ increases or the customers become more willing to return products (*k* becomes larger), the supply chain in Scenario R is more efficient (c.f., the Efficiency of Scenario M is always 75%).

Table 4. Supply Chain Efficiency in Scenario R

Efficiency	k=0. 1	k=0. 3	k=0. 5	k=1. 0	k=1. 5	k=2. 0	k=2. 5
Δ=3	90. 15%	90. 64%	91. 34%	91. 40%	91. 25%	91. 18%	91. 13%
Δ=2	90. 02%	90. 23%	90. 45%	91. 01%	90. 86%	90. 79%	90. 74%
Δ=1	89. 94%	89. 99%	90. 05%	90. 18%	90. 35%	90. 44%	90. 40%

Last, we look at the percentage of the total supply chain profit extracted by the Stackelberg leader in each scenario. In Scenario M, we know that $\Pi^M = 2\Pi^R = 2\Pi^T/3$ from Table 2, or the manufacturer (the leader) extracts 66.7% of the total supply chain profit. In Scenario R, the percentage of the total supply chain profit that the retailer (the leader) obtains in the numerical examples with different parameter values is listed in Table 5. We observe that the retailer in Scenario R extracts relatively more than the manufacturer in Scenario M (average 87.6% in Scenario R vs. 66.7% in Scenario M). Considering both the supply chain efficiency and the percentage of each party, we can conclude that under the same circumstance the retailer in Scenario R can make much more profit than the manufacturer in Scenario M.

Table 5. The Retailer (the Stackelberg Leader) Profit Percentage in Scenario R

Retailer Profit %	k=0.1	k=0.3	k=0.5	k=1.0	k=1.5	k=2.0	k=2.5
Δ=3	86.98%	87.88%	88.65%	88.60%	88.59%	88.58%	88.57%
Δ=2	86.75%	87.12%	87.51%	87.89%	87.88%	87.87%	87.86%
Δ=1	86.62%	86.71%	86.80%	87.03%	87.22%	87.21%	87.21%

7.2 Supply Chain Coordination

In the above analysis, we assume that the centrally coordinated supply chain is automatically achieved. However, unless the manufacturer and the retailer are of the same company or vertically integrated so that the total supply chain profit is the objective, the manufacturer and the retailer could not achieve the coordination automatically. Usually the Stackelberg leader, because of the dominant supply chain power, could initiate a coordination mechanism so that both parties could get better or at least not worse profit than in the uncoordinated supply chain. In this section, we give a description of the possible coordination mechanism available to the Stackelberg leader in the supply chain. We present the analysis as follows.

First, in the retailer-driven supply chain (Scenario R), the retailer's price multipliers β and λ at (β=1, λ=1) have a special meaning in Scenario R. When the retail sets (β=1, λ=1), the retailer's price will be $\prod^R = D(p)[p-w+r(c)(b-c)] = D(w\beta)[w\beta-w+r(b\lambda)(b-b\lambda)]=0$; the manufacturer's profit will be $\prod^M = D(p)[w-cm+r(c)(\Delta-b)] = D(w)[w-cm+r(b)(\Delta-b)]$. To maximize this manufacturer's profit $\prod^M$ over w and b is equivalent to maximizing the total supply chain profit $\prod^T$ over p and c in Scenario C, and we will get ($p=w=p_C^*$, $b=c=c_C^*$). Therefore, (β=1, λ=1) is a sufficient condition for $\prod^T$ maximization in Scenario R. Because of the specific structure of the objection function, (β=1, λ=1) is also a necessary condition for $\prod^T$ maximization in Scenario R (Proof see Appendix X).

In the manufacturer-driven supply chain (Scenario M), the manufacturer's prices w and b at ($w=cm$, $b=\Delta$) have a special meaning. When the manufacturer sets ($w=cm$, $b=\Delta$), i.e., when the manufacturer's profit $\prod^M = D(p)[w-cm+r(c)(\Delta-b)]=0$, the retailer's profit is $\prod^R = D(p)[p-w+r(c)(b-c)] = D(p)[p-cm+r(c)(\Delta-c)]$. To maximize this retailer's profit $\prod^R$ over p and c is equivalent to maximizing the total supply chain profit $\prod^T$ over p and c in Scenario C, and we will get ($p=p_C^*$, $c=c_C^*$). Therefore, ($w=cm$, $b=\Delta$) is a sufficient condition for $\prod^T$ maximization in Scenario M. Because of the specific structure of the objection function, ($w=cm$, $b=\Delta$) is also a necessary condition for $\prod^T$ maximization in Scenario M (Proof see Appendix IX).

Based on the above discussion, we briefly discuss the coordination mechanism for the supply chain in Scenario M and Scenario R. Specially, we assume that the retailer will initiate in Scenario R and the manufacturer will initiate the coordination in Scenario M because of the Stackelberg leader position.

In Scenario R, the two parties can adopt any of the following two effective coordination mechanisms:

(R1) The retailer sets (β=1, λ=1) and the manufacturer pays a fixed payment to shift a part of the total supply chain profit to the retailer. In this way, the retailer acts a selling and collecting agent and is compensated by a fixed lump sum payment.

(R2) The manufacturer and the retailer have an implicit understanding (see Jeuland and Shugan, 1983) about the setting of p and c to p_C^* and c_C^* to achieve maximal total supply chain profit. The retailer then set β>1 or λ<1 and require the wholesale price $w = p_C^* / \beta$ or $b = c_C^* / \lambda$ from the manufacturer.

Similarly, in Scenario M, the two parties can adopt any of the following two effective coordination mechanisms:

(M1) The manufacturer sets (w=cm, b=Δ) and the retailer pays a fixed payment to shift a part of the total supply chain profit to the manufacturer. In this way, the manufacturer acts a manufacturing and remanufacturing agent and is compensated by a fixed lump sum payment.

(M2) The manufacturer and the retailer have an implicit understanding (see Jeuland and Shugan, 1983) about the setting of p and c to p_C^* and c_C^* to achieve maximal total supply chain profit. The manufacturer then sets w>cm or b<Δ to obtain a part of the total supply chain profit while leaving the remaining to the retailer.

The exact amount of the fixed payment in (R1) or (M1) and the exact values of β and λ in (R2) or the exact prices w and b in (M2) are an important negotiation issue in coordination. The determination of these values introduces the quantitative measure of the supply chain power, which is beyond the scope of this paper. However, what remains always true is that the retail price p and the collecting price c will be set to the optimal point ($p = p_C^*$, $c = c_C^*$), which is the link between the retailer-driven or manufacturer-driven model to the centrally coordinated model. We also note that the coordination mechanism is subject to the pricing laws that may affect the implementation of these coordination mechanisms.

7.3 Policy Implications of the Models

Based on the analysis and numerical examples above, we summarize the price decision in the three scenarios in the following Table 6.

Table 6. Comparison of Three Scenarios

	From Analysis	From Numerical Example
w^*		$w_M^* > w_R^*$ (♀)
b^*	$b_M^* = \Delta > b_R^*$	
p^*	$p_M^* > p_C^*$ when $2D'(p)^2 > D(p)D''(p)$	$p_C^* < p_R^* < p_M^*$ (♀)
c^*	$c_M^* = c_C^*$	$c_C^* = c_M^* < c_R^*$ (♀)
Π^M		$\Pi_M^M > \Pi_R^M$ (♀)
Π^R		$\Pi_M^R < \Pi_R^R$ (♀)
Π^T		$\Pi_M^T < \Pi_R^T$ (♀)

(♀) From Numerical Examples

Besides the many managerial insights discussed in above sections, the most interesting implication for decision makers is that the retailer-driven supply chain results in higher return rate of used product than the manufacturer-driven supply chain and the centrally coordinated supply chain (from $c_C^* = c_M^* < c_R^*$ and thus $r(c_C^*) = r(c_M^*) < r(c_R^*)$). This means that if the remanufacturing (because of environmental concerns) is of paramount importance, the retailer-driven supply chain may be the most recommended situation, even better than the centrally coordinated supply chain. Decision/policy support for community may prefer to attract Retailer-driven over Manufacturer-driven or Centrally Coordinated supply chain with subsidies, tax reduction, etc. However, the concrete steps to achieve these policy decisions need further study.

Another interesting implication is that from the consumer's point of view, the centrally coordinated supply chain could generate the largest consumer surplus, because $p_C^* < p_R^* < p_M^*$, i.e., the consumers would prefer the centrally coordinated supply chain to the other two alternatives. Policy makers who seek maximal consumer surplus would recommend the manufacturer and the retailer to coordinate in the supply chain.

7.4 Third Party Collecting Model

The retailer-driven and the manufacturer-driven model discussed above are basic models. More complicated supply chain models with different supply chain structure could be developed based on these basic models.

As one simple example, in this section, we introduce a third party as the collecting agent in the supply chain. Savaskan (2004) discusses a steady state supply chain model where the manufacturer is the Stackelberg leader and a 3rd party collects the used product (see Figure 11). Under their assumptions about the collecting cost and return rate (i.e., $C(\tau) = I + A\tau D(p) = C_L\tau^2 + A\tau D(p)$, see Chapter 5 for detailed description of the function), the major findings of their model is that the inclusion of the 3rd party in the supply chain increases the retail price and decreases return rate, i.e., the inclusion of the 3rd party is not recommended for the supply chain purely from the perspective of the profits and the remanufacturing.

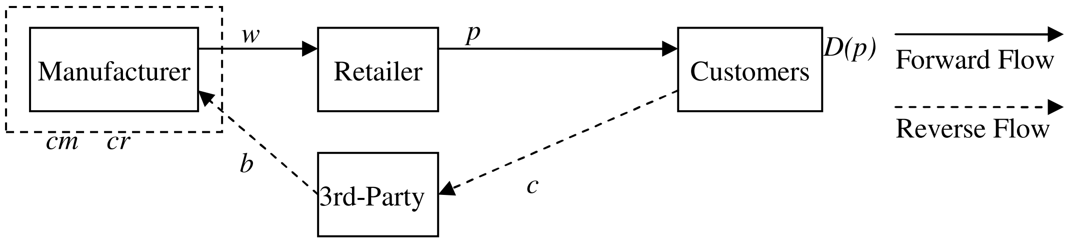


Figure 11. 3rd Party Collecting, Manufacturer-driven

On the other hand, the supply chain model with retailer as the Stackelberg leader and the 3rd party as the collecting agent (see Figure 12) was not investigated. We have the same assumption as that in the basic retailer-driven model. Also we assume that the profit of the 3rd party is $\Pi^{3P} = D(p)r(c)(d-c)$. The solution of this retailer-driven 3rd party supply chain model could be easily obtained from the basic retailer-driven model discussed in the paper, by making some function transformations, i.e., changing the collecting function from *r(c)* to *r(d)* = *r(c*(d))*, where *c*(d)*= $\arg\underset{c}{Max}\,\Pi^{3P}$. Then we could follow the same solution procedure discussed in the Chapter 4 to derive a solution for the retailer-driven 3rd party collecting model. We note the SOSC's can be derived to check the optimality as in Chapter 3.

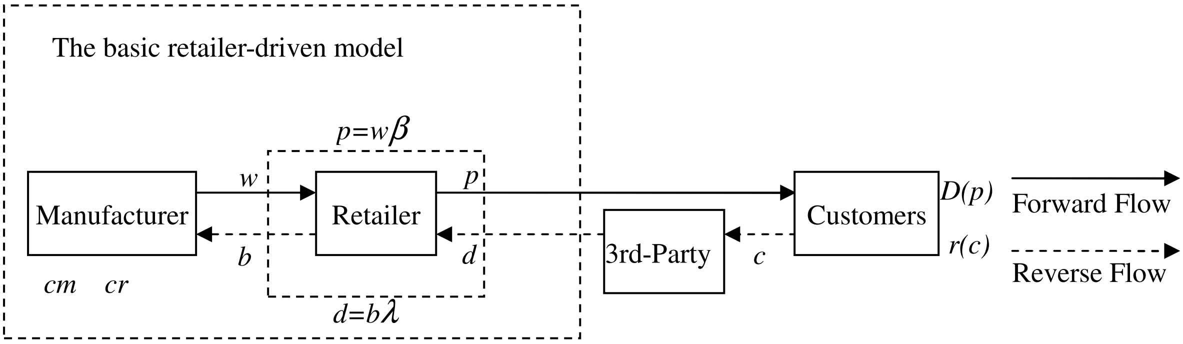


Figure 12. 3rd Party Collecting, Retailer-driven

Specifically, in the case of linear return rate function ($r(c)=kc$), we have $\Pi^{3P} = D(p)r(c)(d\text{-}c) = D(p)kc(d\text{-}c)$. To maximize Π^{3P}, we have $c^*(d)=d/2$. So we have $r(d) = r(d/2)=kd/2= (k/2)d$, which means that the retailer-driven 3rd party collecting model is equivalent to the basic retailer-driven (retailer-collecting) with half of the original k.

From the above analysis, the impact of introducing the 3rd collecting party in the supply chain is equivalent to the case that the original k will be replaced by $k/2$ in the basic model, and the solution to the basic model is the solution of the retailer-driven 3rd party collecting model. Because the k is reduced to half, the impact is that the optimal λ is reduced if the return rate $r(c)<1$. Refer to Figure 4 to see the impact of the reduced k.

CHAPTER 8. SUMMARY AND FUTURE WORK

8.1 Summary of the Paper

In this paper, we studied the pricing strategy of a basic two-echelon closed-loop market-driven remanufacturing supply chain in different supply chain power structures, i.e., retailer-driven supply chain, centrally coordinated supply chain, the manufacturer-driven supply chain. We found that that the Stackelberg leader in the supply chain adopts a loss leader pricing policy towards the reverse flow by giving up the profit from the reverse flow and extracting the maximal profit from the forward flow only.

In the retailer-driven model, we found that retailer sets the reverse flow price multiplier $\lambda>1$ to help the manufacturer to collect more used products. We showed the optimality analysis of the basic retailer-driven (retailer-collecting) model and the retailer-driven manufacturer-collecting model at $\lambda=1$. Through numerical example, we showed that this loss leader pricing strategy is prevailing in retailer pricing decision.

In the manufacturer-driven model, we demonstrate that the manufacturer sets $b_M^* = \Delta$ to give up all the profit in the reverse flow. By doing this, the manufacturer could induce the retailer to reduce the retail price and thus increase demand, which make the manufacturer extract the maximal profit in the forward flow.

We then discussed other issues of the models. We compared the retailer-driven model and the manufacturer-driven model and found that retailer-driven model has more efficiency and the retailer could extract more profit in terms of the percentage of the total supply chain profit. This advantage of the retailer coincides with the findings in many literatures and is attributed to being closer to customers.

We also showed two coordination mechanisms available to the Stackelberg leader in the supply chain: lump sum payment or an implicit understanding.

For the policy implication, we showed that retailer-driven supply chain is preferred if maximizing the return rate of the used product is sought; whereas for maximization of the consumer surplus, the coordinated supply chain is the best.

As an example of extending the basic models to more complicated models with different supply chain structure, a third party collecting model is briefly discussed.

Some other interesting findings are also presented throughout the paper, e.g., the retailer-driven manufacturer-collecting model, the linear model solutions, the optimal return price b decision in retailer-driven model, etc.

8.2 Future Work

Because there are few research literatures regarding the super retailer and the remanufacturing practice in the supply chain, we believe there are many interesting open research area.

For example, this paper is about the supply chain with remanufacturing in a steady state. A natural extension of this research is to investigate the evolution of the supply chain in a dynamic process from some initial state to the steady state. The equilibrium analysis of the steady state, e.g., the existence and uniqueness of the equilibrium, is also interesting. A multi-period discrete model or a continuous model of the supply chain is needed to model the process.

The supply chain with remanufacturing could also contain multiple parties with different gaming process, e.g., two or more competing manufacturers/remanufacturers, two or more retailers, multi-echelon supply chain. The combination is endless and all these models could be developed from the basic model discussed in this paper. It will be interesting to know whether the loss leader pricing policy is still observed in these extended models.

APPENDIX I. The Intermediate Solution of Max $\prod^R$ for the case of *r(c)=1*

If the optimal c^*>1/k from the optimization of $\prod^R$ in section 3.4, then that means that the return rate is within the infeasible region (i.e., $r(c^*)=kc^*>1$) and we need to adjust the solution to the bounded solution. Since the retailer has supply chain power, she will keep the manufacturer paying the return price $b^* \equiv \Delta/2$ while she lowers λ^* to achieve $r(c)$=1, i.e., to keep the return rate $r(c^*)= kc^*= k\lambda^* b^*=1$, the optimal λ^* will be set as $\lambda^*=2/(\Delta k)$. The manufacturer's reaction function under $\lambda=\lambda^*=2/(\Delta k)$ can be found as follows

$$w^*(\beta,\lambda) = [cm + cr + 2u/(v\beta)]/4 \quad \text{and} \quad b^*(\beta,\lambda) = \Delta/2$$

Notice that $w^*(\beta,\lambda)$ depends on β only and $b^*(\beta,\lambda)$ is actually independent on β and λ. The retailer's problem is then

$$Max\,\Pi^R(\beta) = \frac{[2u-(cm+cr)v\beta]\{k[2u(\beta-1)+v\beta(cm-3cr+(cm+cr)\beta)]-4v\beta\}}{16kv\beta} \quad \text{(I.1)}$$

Expression (I.1) is a univariate polynomial and the optimal β^* can be found using many optimization methods. After obtaining β^* from (I.1) and considering $\lambda^*=2/(\Delta k)$, $b^*=\Delta/2$, we can then find optimal $w^*=w^*(\beta,\lambda)$, $p^*=\beta^* w^*$, $c^*=\lambda^* b^*$=1/k and the profit of each party and the total supply chain.

From the intermediate solution of section 3.4 and this appendix, we can find that the manufacturer will always set return price $b^*=\Delta/2$, no matter whether the return rate hits 1 or not. Also when the return rate hits 1, the retailer actually lowers the price multiplier λ to take all the savings.

APPENDIX II. Discussion of Maximization of Expression (6)

It is difficult to derive a closed-form solution from maximization of Expression (6). For Expression (6) by solving $\partial\prod^R/\partial\beta=0$, we obtain the optimal β as a function of λ (after deleting the other two imaginary roots) is

$$
\begin{aligned}
\beta^* = {} & \frac{16\, cm^2 v^2 - 16\, cm\, k\, v^2 \Delta^2 \lambda + 8\, cm\, k\, v^2 \Delta^2 \lambda^2 + 3 k^2 v^2 \Delta^4 \lambda^2 - 2 k^2 v^2 \Delta^4 \lambda^3}{6\,(16\, cm^2 v^2 - 8\, cm\, k\, v^2 \Delta^2 \lambda + k^2 v^2 \Delta^4 \lambda^2)} - \\
& \left(16\, cm^2 v^2 - 16\, cm\, k\, v^2 \Delta^2 \lambda + 8\, cm\, k\, v^2 \Delta^2 \lambda^2 + 3 k^2 v^2 \Delta^4 \lambda^2 - 2 k^2 v^2 \Delta^4 \lambda^3\right)^2 / \\
& \Big(3\, 2^{2/3} \left(16\, cm^2 v^2 - 8\, cm\, k\, v^2 \Delta^2 \lambda + k^2 v^2 \Delta^4 \lambda^2\right) \\
& \quad \big(-442368\, cm^4 u^2 v^4 - 8192\, cm^6 v^6 + 442368\, cm^3 k\, u^2 v^4 \Delta^2 \lambda + 24576\, cm^5 k\, v^6 \Delta^2 \lambda - 12288\, cm^5 k\, v^6 \Delta^2 \lambda^2 - \\
& \quad 165888\, cm^2 k^2 u^2 v^4 \Delta^4 \lambda^2 - 29184\, cm^4 k^2 v^6 \Delta^4 \lambda^2 + 27648\, cm^4 k^2 v^6 \Delta^4 \lambda^3 + 27648\, cm\, k^3 u^2 v^4 \Delta^6 \lambda^3 + \\
& \quad 17408\, cm^3 k^3 v^6 \Delta^6 \lambda^3 - 6144\, cm^4 k^2 v^6 \Delta^4 \lambda^4 - 23040\, cm^3 k^3 v^6 \Delta^6 \lambda^4 - 1728 k^4 u^2 v^4 \Delta^8 \lambda^4 - \\
& \quad 5472\, cm^2 k^4 v^6 \Delta^8 \lambda^4 + 9216\, cm^3 k^3 v^6 \Delta^6 \lambda^5 + 8832\, cm^2 k^4 v^6 \Delta^8 \lambda^5 + 864\, cm\, k^5 v^6 \Delta^{10} \lambda^5 - \\
& \quad 1024\, cm^3 k^3 v^6 \Delta^6 \lambda^6 - 4608\, cm^2 k^4 v^6 \Delta^8 \lambda^6 - 1584\, cm\, k^5 v^6 \Delta^{10} \lambda^6 - 54 k^6 v^6 \Delta^{12} \lambda^6 + 768\, cm^2 k^4 v^6 \Delta^8 \lambda^7 + \\
& \quad 960\, cm\, k^5 v^6 \Delta^{10} \lambda^7 + 108 k^6 v^6 \Delta^{12} \lambda^7 - 192\, cm\, k^5 v^6 \Delta^{10} \lambda^8 - 72 k^6 v^6 \Delta^{12} \lambda^8 + 16 k^6 v^6 \Delta^{12} \lambda^9 + \\
& \quad \surd\Big(-4 \left(16\, cm^2 v^2 - 16\, cm\, k\, v^2 \Delta^2 \lambda + 8\, cm\, k\, v^2 \Delta^2 \lambda^2 + 3 k^2 v^2 \Delta^4 \lambda^2 - 2 k^2 v^2 \Delta^4 \lambda^3\right)^6 + \\
& \qquad \big(-442368\, cm^4 u^2 v^4 - 8192\, cm^6 v^6 + 442368\, cm^3 k\, u^2 v^4 \Delta^2 \lambda + 24576\, cm^5 k\, v^6 \Delta^2 \lambda - \\
& \qquad 12288\, cm^5 k\, v^6 \Delta^2 \lambda^2 - 165888\, cm^2 k^2 u^2 v^4 \Delta^4 \lambda^2 - 29184\, cm^4 k^2 v^6 \Delta^4 \lambda^2 + 27648\, cm^4 k^2 v^6 \Delta^4 \lambda^3 + \\
& \qquad 27648\, cm\, k^3 u^2 v^4 \Delta^6 \lambda^3 + 17408\, cm^3 k^3 v^6 \Delta^6 \lambda^3 - 6144\, cm^4 k^2 v^6 \Delta^4 \lambda^4 - 23040\, cm^3 k^3 v^6 \Delta^6 \lambda^4 - \\
& \qquad 1728 k^4 u^2 v^4 \Delta^8 \lambda^4 - 5472\, cm^2 k^4 v^6 \Delta^8 \lambda^4 + 9216\, cm^3 k^3 v^6 \Delta^6 \lambda^5 + 8832\, cm^2 k^4 v^6 \Delta^8 \lambda^5 + \\
& \qquad 864\, cm\, k^5 v^6 \Delta^{10} \lambda^5 - 1024\, cm^3 k^3 v^6 \Delta^6 \lambda^6 - 4608\, cm^2 k^4 v^6 \Delta^8 \lambda^6 - 1584\, cm\, k^5 v^6 \Delta^{10} \lambda^6 - \\
& \qquad 54 k^6 v^6 \Delta^{12} \lambda^6 + 768\, cm^2 k^4 v^6 \Delta^8 \lambda^7 + 960\, cm\, k^5 v^6 \Delta^{10} \lambda^7 + 108 k^6 v^6 \Delta^{12} \lambda^7 - \\
& \qquad 192\, cm\, k^5 v^6 \Delta^{10} \lambda^8 - 72 k^6 v^6 \Delta^{12} \lambda^8 + 16 k^6 v^6 \Delta^{12} \lambda^9\big)^2\Big)\big)^{1/3}\Big) - \\
& \big(-442368\, cm^4 u^2 v^4 - 8192\, cm^6 v^6 + 442368\, cm^3 k\, u^2 v^4 \Delta^2 \lambda + 24576\, cm^5 k\, v^6 \Delta^2 \lambda - 12288\, cm^5 k\, v^6 \Delta^2 \lambda^2 - \\
& \quad 165888\, cm^2 k^2 u^2 v^4 \Delta^4 \lambda^2 - 29184\, cm^4 k^2 v^6 \Delta^4 \lambda^2 + 27648\, cm^4 k^2 v^6 \Delta^4 \lambda^3 + 27648\, cm\, k^3 u^2 v^4 \Delta^6 \lambda^3 + \\
& \quad 17408\, cm^3 k^3 v^6 \Delta^6 \lambda^3 - 6144\, cm^4 k^2 v^6 \Delta^4 \lambda^4 - 23040\, cm^3 k^3 v^6 \Delta^6 \lambda^4 - 1728 k^4 u^2 v^4 \Delta^8 \lambda^4 - \\
& \quad 5472\, cm^2 k^4 v^6 \Delta^8 \lambda^4 + 9216\, cm^3 k^3 v^6 \Delta^6 \lambda^5 + 8832\, cm^2 k^4 v^6 \Delta^8 \lambda^5 + 864\, cm\, k^5 v^6 \Delta^{10} \lambda^5 - \\
& \quad 1024\, cm^3 k^3 v^6 \Delta^6 \lambda^6 - 4608\, cm^2 k^4 v^6 \Delta^8 \lambda^6 - 1584\, cm\, k^5 v^6 \Delta^{10} \lambda^6 - 54 k^6 v^6 \Delta^{12} \lambda^6 + 768\, cm^2 k^4 v^6 \Delta^8 \lambda^7 + \\
& \quad 960\, cm\, k^5 v^6 \Delta^{10} \lambda^7 + 108 k^6 v^6 \Delta^{12} \lambda^7 - 192\, cm\, k^5 v^6 \Delta^{10} \lambda^8 - 72 k^6 v^6 \Delta^{12} \lambda^8 + 16 k^6 v^6 \Delta^{12} \lambda^9 + \\
& \quad \surd\Big(-4 \left(16\, cm^2 v^2 - 16\, cm\, k\, v^2 \Delta^2 \lambda + 8\, cm\, k\, v^2 \Delta^2 \lambda^2 + 3 k^2 v^2 \Delta^4 \lambda^2 - 2 k^2 v^2 \Delta^4 \lambda^3\right)^6 + \\
& \qquad \big(-442368\, cm^4 u^2 v^4 - 8192\, cm^6 v^6 + 442368\, cm^3 k\, u^2 v^4 \Delta^2 \lambda + 24576\, cm^5 k\, v^6 \Delta^2 \lambda - \\
& \qquad 12288\, cm^5 k\, v^6 \Delta^2 \lambda^2 - 165888\, cm^2 k^2 u^2 v^4 \Delta^4 \lambda^2 - 29184\, cm^4 k^2 v^6 \Delta^4 \lambda^2 + 27648\, cm^4 k^2 v^6 \Delta^4 \lambda^3 + \\
& \qquad 27648\, cm\, k^3 u^2 v^4 \Delta^6 \lambda^3 + 17408\, cm^3 k^3 v^6 \Delta^6 \lambda^3 - 6144\, cm^4 k^2 v^6 \Delta^4 \lambda^4 - 23040\, cm^3 k^3 v^6 \Delta^6 \lambda^4 - \\
& \qquad 1728 k^4 u^2 v^4 \Delta^8 \lambda^4 - 5472\, cm^2 k^4 v^6 \Delta^8 \lambda^4 + 9216\, cm^3 k^3 v^6 \Delta^6 \lambda^5 + 8832\, cm^2 k^4 v^6 \Delta^8 \lambda^5 + \\
& \qquad 864\, cm\, k^5 v^6 \Delta^{10} \lambda^5 - 1024\, cm^3 k^3 v^6 \Delta^6 \lambda^6 - 4608\, cm^2 k^4 v^6 \Delta^8 \lambda^6 - 1584\, cm\, k^5 v^6 \Delta^{10} \lambda^6 - \\
& \qquad 54 k^6 v^6 \Delta^{12} \lambda^6 + 768\, cm^2 k^4 v^6 \Delta^8 \lambda^7 + 960\, cm\, k^5 v^6 \Delta^{10} \lambda^7 + 108 k^6 v^6 \Delta^{12} \lambda^7 - 192\, cm\, k^5 v^6 \Delta^{10} \lambda^8 - \\
& \qquad 72 k^6 v^6 \Delta^{12} \lambda^8 + 16 k^6 v^6 \Delta^{12} \lambda^9\big)^2\Big)\big)^{1/3} \Big/ \left(6\, 2^{1/3} \left(16\, cm^2 v^2 - 8\, cm\, k\, v^2 \Delta^2 \lambda + k^2 v^2 \Delta^4 \lambda^2\right)\right)
\end{aligned}
$$

By solving $\partial\prod^R/\partial\lambda=0$, we obtain the optimal λ as a function of β is

$$
\lambda^* = -\frac{1}{6 k v \beta \Delta^2}\left(8u - 8\, cm\, v\beta - 3 k v \beta \Delta^2 + k v \beta^2 \Delta^2 \pm \sqrt{12 k v \beta\,(4u - 8\, cm\, v\beta + 4\, cm\, v\beta^2)\,\Delta^2 + (-8u + 8\, cm\, v\beta + 3 k v \beta \Delta^2 - k v \beta^2 \Delta^2)^2}\right)
$$

We find it is analytically difficult to solve $\{\partial\prod^R/\partial\beta=0,\ \partial\prod^R/\partial\lambda=0\}$, and yield meaningful insights.

APPENDIX III. Proof of $\left.\frac{\partial \Pi^{R}(\lambda,\beta)}{\partial \lambda}\right|_{(\lambda=1,\beta>\beta^{1})} > 0$ **in the retailer-driven model**

For the case r(c)<1, from Expression (6), we have

$$\Pi^{R}(\beta,\lambda)=\frac{[4(\beta-1)(u+cm\,v\beta)-kv\beta\lambda\Delta^{2}(\beta+2\lambda-3)][4u+v\beta(k\lambda\Delta^{2}-4cm)]}{64v\beta}$$

$$\frac{\partial \Pi^{R}(\lambda,\beta)}{\partial \lambda}=k\Delta^{2}[u(4-8\lambda)+v\beta(4cm(\beta+2\lambda-2)-k\Delta^{2}\lambda(\beta+3\lambda-3))]/32$$

$$\left.\frac{\partial \Pi^{R}(\lambda,\beta)}{\partial \lambda}\right|_{(\lambda=1,\beta)}=k\Delta^{2}[-4u+v\beta^{2}(4cm-k\Delta^{2})]/32$$

The equation $\left.\frac{\partial \Pi^{R}(\lambda,\beta)}{\partial \lambda}\right|_{(\lambda=1,\beta)}=0$ has two roots $\beta=\pm\frac{2\sqrt{u}}{\sqrt{v(4cm-k\Delta^{2})}}$. Let $\beta^{l}=\frac{2\sqrt{u}}{\sqrt{v(4cm-k\Delta^{2})}}$. It is apparent that $\left.\frac{\partial \Pi^{R}(\beta,\lambda)}{\partial \lambda}\right|_{(\beta=\beta^{1},\lambda=1)}=0$ and $\left.\frac{\partial \Pi^{R}(\beta,\lambda)}{\partial \lambda}\right|_{(\beta>\beta^{1},\lambda=1)}>0$.■

APPENDIX IV. Proof of $\beta^2 > \beta^1$ in the RDMC model.

For the case of *r(c)*<1, from Expression (6), we have

$$
\begin{aligned}
\Pi^{R}_{RDMC}(\beta) = \Pi^{R}(\beta, \lambda = 1) &= \frac{(\beta - 1)[4u + v\beta(4cm - k\Delta^2)][4u + v\beta(k\Delta^2 - 4cm)]}{64 v\beta} \\
&= \frac{-u^2}{4v\beta} - \frac{\beta^2 v(4cm - k\Delta^2)^2}{64} + \frac{u^2}{4v} + \frac{v\beta(4cm - k\Delta^2)^2}{64}
\end{aligned}
$$

Apparently, $\lim_{\beta \to 0} \Pi^{R}_{RDMC}(\beta) = -\infty$ and $\lim_{\beta \to \infty} \Pi^{R}_{RDMC}(\beta) = -\infty$ (when $4\,cm - k\Delta^2 > 0$).

By solving

$$
\frac{\partial \Pi^{R}_{RDMC}(\beta)}{\partial \beta} = \frac{16u^2 - v^2\beta^2(2\beta - 1)(4cm - k\Delta^2)^2}{64 v\beta^2} = 0
$$

We find that there are one real root and two imaginary roots.

The real root is

$$
\begin{aligned}
\beta^* = {} & \\
& \frac{1}{6} - \left(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4\right) / \\
& \Big(3\, 2^{2/3} \\
& \quad \big(-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + \\
& \quad\quad 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12} + \\
& \quad\quad \sqrt{}\Big(-4\left(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4\right)^6 + \\
& \quad\quad\quad \big(-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + \\
& \quad\quad\quad\quad 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12}\big)^2\Big)\big)^{1/3}\Big) - \\
& \big(-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + \\
& \quad 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12} + \\
& \quad \sqrt{}\Big(-4\left(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4\right)^6 + \\
& \quad\quad \big(-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + \\
& \quad\quad\quad 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12}\big)^2\Big)\big)^{1/3} / \\
& \left(6\, 2^{1/3} \left(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4\right)\right)
\end{aligned}
$$

Two imaginary roots are

$$\beta^* = \frac{1}{6} + \Big((1 + i\sqrt{3})\,(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4)\Big) \Big/ \Big(6\,2^{2/3}\,\big(-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12} + \sqrt{\big(-4\,(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4)^6 + (-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12})^2\big)}\big)^{1/3}\Big) + \Big((1 - i\sqrt{3})\,\big(-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12} + \sqrt{\big(-4\,(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4)^6 + (-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12})^2\big)}\big)^{1/3}\Big) \Big/ \Big(12\,2^{1/3}\,(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4)\Big)$$

$$\beta^* = \frac{1}{6} + \Big((1 - i\sqrt{3})\,(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4)\Big) \Big/ \Big(6\,2^{2/3}\,\big(-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12} + \sqrt{\big(-4\,(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4)^6 + (-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12})^2\big)}\big)^{1/3}\Big) + \Big((1 + i\sqrt{3})\,\big(-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12} + \sqrt{\big(-4\,(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4)^6 + (-442368\,cm^4 u^2 v^4 - 8192\,cm^6 v^6 + 442368\,cm^3 k\,u^2 v^4 \Delta^2 + 12288\,cm^5 k\,v^6 \Delta^2 - 165888\,cm^2 k^2 u^2 v^4 \Delta^4 - 7680\,cm^4 k^2 v^6 \Delta^4 + 27648\,cm\,k^3 u^2 v^4 \Delta^6 + 2560\,cm^3 k^3 v^6 \Delta^6 - 1728\,k^4 u^2 v^4 \Delta^8 - 480\,cm^2 k^4 v^6 \Delta^8 + 48\,cm\,k^5 v^6 \Delta^{10} - 2\,k^6 v^6 \Delta^{12})^2\big)}\big)^{1/3}\Big) \Big/ \Big(12\,2^{1/3}\,(16\,cm^2 v^2 - 8\,cm\,k\,v^2 \Delta^2 + k^2 v^2 \Delta^4)\Big)$$

Because there is only one real root and $\lim_{\beta \to 0} \Pi^{R}_{RDMC}(\beta) = -\infty$ and $\lim_{\beta \to \infty} \Pi^{R}_{RDMC}(\beta) = -\infty$, $\Pi^{R}_{RDMC}(\beta)$ is concave in $\beta (\beta > 0)$ and a typical $\Pi^{R}_{RDMC}(\beta)$ looks like in Figure 13.

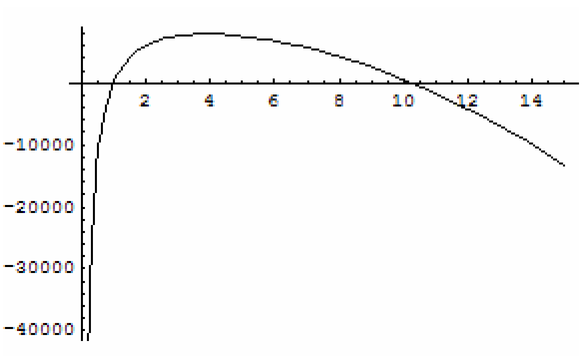


Figure 13. Retailer's Profit Function in RDMC Model

On the other hand, the derivative of $\Pi^{R}_{RDMC}(\beta)$ at $\beta^{1}=\dfrac{2\sqrt{u}}{\sqrt{v(4cm-k\Delta^{2})}}$ is

$$\left.\frac{\partial\Pi^{R}_{RDMC}(\beta)}{\partial\beta}\right|_{(\beta=\beta^{1})}=(4cm-k\Delta^{2})[4cm-k\Delta^{2}+4u-4\sqrt{u}\sqrt{v(4cm-k\Delta^{2})}]/64$$

It is apparent that if $4cm-k\Delta^{2}>0$ and $u>v(4cm-k\Delta^{2})$ (both inequalities are reasonable assumptions for practical parameter values because k is usually a small number, and u is usually a big number), then $\left.\dfrac{\partial\Pi^{R}_{RDMC}(\beta)}{\partial\beta}\right|_{(\beta=\beta^{1})}>0$. Therefore, the optimal solution of $Max\,\Pi^{R}_{RDMC}(\beta)$ is obtain at a point β^{2}, which satisfies $\beta^{2}>\beta^{1}$.■

APPENDIX V. Numerical Examples of Retailer-driven Model

Numerical example with different *k* (*k*=0.05 to 1.6) and Δ (Δ=1,2,3) values:

Δ=1				
k	r(c)	λ	β	Retailer's Profit
0.05	0.035	1.3987	3.8676	7889.217
0.1	0.07	1.3994	3.8767	7896.738
0.15	0.105	1.4002	3.886	7904.273
0.2	0.1401	1.4009	3.8953	7911.821
0.25	0.1752	1.4017	3.9047	7919.383
0.3	0.2104	1.4024	3.9141	7926.958
0.35	0.2456	1.4032	3.9236	7934.548
0.4	0.2808	1.404	3.9332	7942.151
0.45	0.3161	1.4048	3.9428	7949.768
0.5	0.3514	1.4055	3.9526	7957.399
0.55	0.3867	1.4063	3.9624	7965.045
0.6	0.4221	1.4071	3.9722	7972.704
0.65	0.4576	1.4079	3.9822	7980.378
0.7	0.493	1.4087	3.9922	7988.066
0.75	0.5286	1.4095	4.0023	7995.769
0.8	0.5641	1.4103	4.0125	8003.486
0.85	0.5997	1.4111	4.0228	8011.218
0.9	0.6354	1.4119	4.0331	8018.964
0.95	0.6711	1.4127	4.0435	8026.725
1	0.7068	1.4136	4.054	8034.501
1.05	0.7426	1.4144	4.0646	8042.292
1.1	0.7784	1.4152	4.0753	8050.098
1.15	0.8142	1.4161	4.0861	8057.919
1.2	0.8501	1.4169	4.0969	8065.756
1.25	0.8861	1.4178	4.1079	8073.608
1.3	0.9221	1.4186	4.1189	8081.475
1.35	0.9581	1.4195	4.13	8089.358
1.4	0.9942	1.4203	4.1412	8097.256
1.45	1	1.3793	4.1413	8104.972
1.5	1	1.3333	4.1395	8112.184
1.55	1	1.2903	4.1377	8118.931
1.6	1	1.25	4.1361	8125.259

Δ=2				
k	r(c)	λ	β	Retailer's Profit
0.05	0.0700	1.4009	3.8953	7911.821
0.10	0.1404	1.4040	3.9332	7942.151
0.15	0.2111	1.4071	3.9722	7972.704
0.20	0.2821	1.4103	4.0125	8003.486
0.25	0.3534	1.4136	4.0540	8034.501
0.30	0.4251	1.4169	4.0969	8065.756
0.35	0.4971	1.4203	4.1412	8097.256
0.40	0.5695	1.4238	4.1870	8129.007
0.45	0.6423	1.4274	4.2344	8161.017
0.50	0.7156	1.4311	4.2834	8193.291
0.55	0.7892	1.4349	4.3343	8225.838
0.60	0.8633	1.4388	4.3870	8258.664
0.65	0.9378	1.4428	4.4418	8291.778
0.70	1.0000	1.4286	4.4872	8325.115
0.75	1.0000	1.3333	4.4785	8355.657
0.80	1.0000	1.2500	4.4709	8382.408
0.85	1.0000	1.1765	4.4642	8406.032
0.90	1.0000	1.1111	4.4583	8427.049
0.95	1.0000	1.0526	4.4530	8445.866
1.00	1.0000	1.0000	4.4483	8462.812
1.05	1.0000	0.9524	4.4440	8478.153
1.10	1.0000	0.9091	4.4401	8492.106
1.15	1.0000	0.8696	4.4366	8504.852
1.20	1.0000	0.8333	4.4333	8516.54
1.25	1.0000	0.8000	4.4303	8527.298
1.30	1.0000	0.7692	4.4276	8537.231
1.35	1.0000	0.7407	4.4250	8546.432
1.40	1.0000	0.7143	4.4227	8554.978
1.45	1.0000	0.6897	4.4205	8562.937
1.50	1.0000	0.6667	4.4184	8570.368
1.55	1.0000	0.6452	4.4165	8577.32
1.60	1.0000	0.6250	4.4147	8583.84

Δ=3				
k	r(c)	λ	β	Retailer's Profit
0.05	0.1054	1.4048	3.9428	7949.768
0.10	0.2118	1.4119	4.0331	8018.964
0.15	0.3194	1.4195	4.1300	8089.358
0.20	0.4282	1.4274	4.2344	8161.017
0.25	0.5384	1.4359	4.3473	8234.018
0.30	0.6502	1.4448	4.4699	8308.446
0.35	0.7635	1.4544	4.6039	8384.397
0.40	0.8787	1.4646	4.7511	8461.981
0.45	0.9960	1.4755	4.9140	8541.324
0.50	1.0000	1.3333	4.8974	8614.244
0.55	1.0000	1.2121	4.8785	8674.048
0.60	1.0000	1.1111	4.8628	8723.976
0.65	1.0000	1.0256	4.8496	8766.288
0.70	1.0000	0.9524	4.8384	8802.602
0.75	1.0000	0.8889	4.8287	8834.109
0.80	1.0000	0.8333	4.8203	8861.705
0.85	1.0000	0.7843	4.8128	8886.074
0.90	1.0000	0.7407	4.8062	8907.751
0.95	1.0000	0.7018	4.8004	8927.16
1.00	1.0000	0.6667	4.7951	8944.638
1.05	1.0000	0.6349	4.7903	8960.459
1.10	1.0000	0.6061	4.7860	8974.85
1.15	1.0000	0.5797	4.7820	8987.995
1.20	1.0000	0.5556	4.7784	9000.049
1.25	1.0000	0.5333	4.7751	9011.142
1.30	1.0000	0.5128	4.7721	9021.386
1.35	1.0000	0.4938	4.7692	9030.874
1.40	1.0000	0.4762	4.7666	9039.686
1.45	1.0000	0.4598	4.7642	9047.894
1.50	1.0000	0.4444	4.7619	9055.556
1.55	1.0000	0.4301	4.7598	9062.725
1.60	1.0000	0.4167	4.7578	9069.447

The sensitivity of the forward price multiplier β with respect to k and Δ

From the numerical examples above, we observe that:

(1) when $r(c)<1$, optimal β increase in k; when $r(c)>1$, optimal β decreases in k;

(2) the bigger the Δ, the less sensitive of optimal β with respect to k.

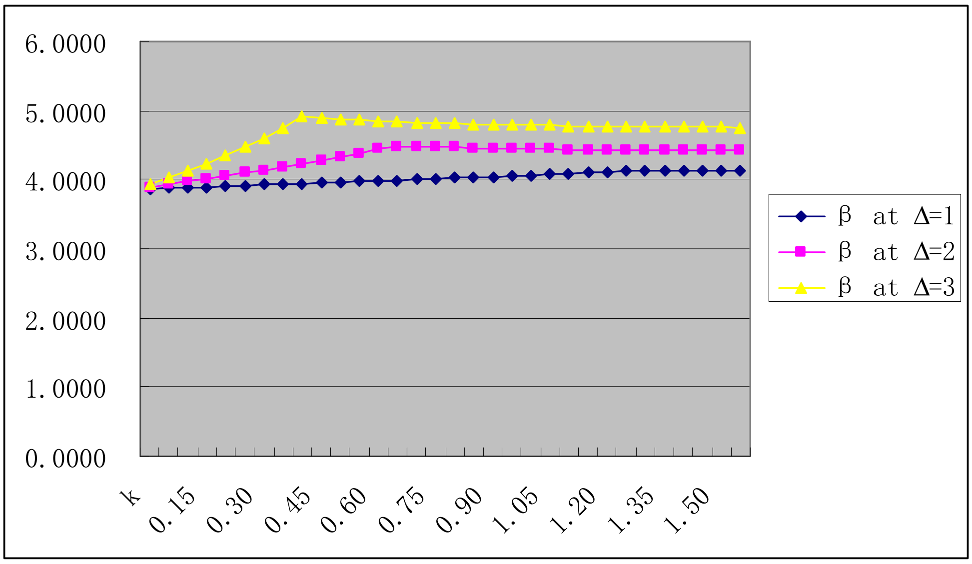
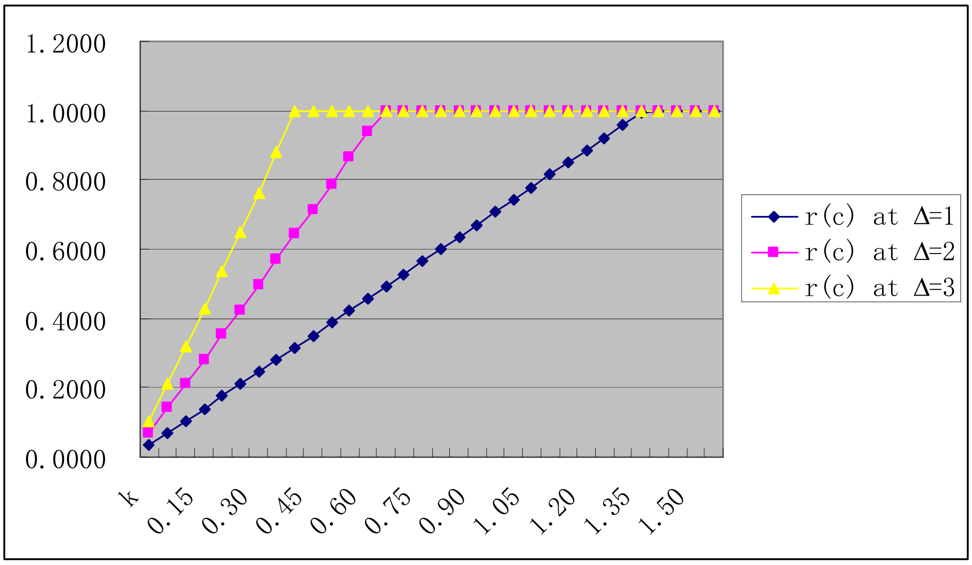


Figure 14. Optimal β vs. k and Δ in Retailer-driven Model



The same Figure as Figure 5. Return Rate $r(c)$ vs. k and Δ in Retailer-driven Model

Numerical example with different *v* values (*v*=5 to 60):

Δ=2, k=0.15				
v	r(c)	λ	β	Retailer's Profit
5	0.2740	1.8269	9.8248	42403.2600
10	0.2391	1.5937	6.2241	19090.8900
15	0.2219	1.4796	4.7809	11598.0600
20	0.2111	1.4071	3.9722	7972.7040
25	0.2033	1.3552	3.4449	5864.7960
30	0.1973	1.3153	3.0694	4502.1820
35	0.1925	1.2833	2.7862	3558.2730
40	0.1885	1.2567	2.5638	2871.7910
45	0.1851	1.2341	2.3836	2354.1980
50	0.1822	1.2145	2.2342	1952.9850
60	0.1773	1.1820	1.9998	1378.7830
70	0.1734	1.1558	1.8232	995.4839
80	0.1701	1.1341	1.6847	727.9104
90	0.1673	1.1155	1.5727	535.1831
100	0.1649	1.0994	1.4799	393.2848
110	0.1628	1.0852	1.4017	287.2462
120	0.1609	1.0726	1.3347	207.2915
130	0.1592	1.0612	1.2767	146.7908
140	0.1576	1.0509	1.2257	101.1045
150	0.1562	1.0414	1.1807	66.8998
160	0.1549	1.0327	1.1406	41.7282
170	0.1537	1.0247	1.1046	23.7557
180	0.1526	1.0173	1.0720	11.5854
190	0.1515	1.0103	1.0425	4.1377
199	0.1507	1.0045	1.0181	0.7697
Δ=2, k=0.4				
v	r(c)	λ	β	Retailer's Profit
10	0.6472	1.6179	6.6177	19318.9400
15	0.5996	1.4990	5.0565	11781.4900
20	0.5695	1.4238	4.1870	8129.0070
25	0.5481	1.3701	3.6221	6002.1390
30	0.5316	1.3290	3.2210	4625.1710
35	0.5184	1.2960	2.9191	3669.8140
40	0.5075	1.2687	2.6823	2973.8610
45	0.4982	1.2454	2.4909	2448.2190
50	0.4901	1.2254	2.3323	2040.0200

APPENDIX VI. Solution Procedure of the Manufacturer-driven Model

For Scenario M, we first derive some features of the retailer's reaction functions; then we maximize the manufacturer's profit by taking into account these reaction functions. The procedure is given as follows.

Proof of Proposition 1. We first set $\partial\prod^R/\partial c=0$ and $\partial\prod^R/\partial p=0$ and find that the optimal c^* and p^* which maximize the retailer's profit $\prod^R$ satisfy the following FONC

$$r'(c)= r(c)/(b-c) \quad \text{(VI.1)}$$

$$D'(p)= -D(p)/[p-w+(b-c)\cdot r(c)] \quad \text{(VI.2)}$$

Because the above optimization is performed under a given pair of b and w, and from (VI.1) and (VI.2), we know that c^* is a function of b, and p^* is a function of b and w. Rewrite (VI.1) and (VI.2) as implicit functions of b and w:

$$r'(c(b))= r(c(b))/(b-c(b)) \quad \text{(VI.3)}$$

$$D'(p(w,b))= -D(p(w,b))/[p(w,b)-w+(b-c(b))\cdot r(c(b))] \quad \text{(VI.4)}$$

By differentiating the two sides of (VI.3) with respect to b, we can find

$$c'(b)= r'(c(b))/[2\, r'(c(b))-(b-c(b))\cdot r''(c(b))] \quad \text{(VI.5)}$$

By differentiating the two sides of (VI.4) with respect to w and b respectively, we can find

$$\frac{\partial p(w,b)}{\partial w}=\frac{D'(p(w,b))}{2D'(p(w,b))+(p(w,b)-w+(b-c(b))\cdot r(c(b)))\cdot D''(p(w,b))} \quad \text{(VI.6)}$$

$$\frac{\partial p(w,b)}{\partial b}=\frac{D'(p(w,b))\cdot[r(c(b))\cdot(c'(b)-1)+(c(b)-b)\cdot c'(b)\cdot r'(c(b))]}{2D'(p(w,b))+(p(w,b)-w+(b-c(b))\cdot r(c(b)))\cdot D''(p(w,b))} \quad \text{(VI.7)}$$

To reflect the reaction function $c^*(b)$ and $p^*(w,b)$ in the manufacturer's profit, we rewrite $\prod^M$ as $\prod^M(w,b)=D(p(w,b))\cdot[w-cm+r(c(b))\cdot(\Delta-b)]$. We then calculate $\partial\prod^M/\partial w$ and

$\partial\prod^{M}/\partial b$ and use (VI.3) to (VI.7) to replace the relevant terms. We find the following

$$\frac{\partial\Pi^{M}}{\partial w}=D(p(w,b))-\frac{D(p(w,b))^{2}(w-cm+(\Delta-b)\cdot r(c(b)))}{X\cdot(-2D(p(w,b))+X^{2}\cdot D''(p(w,b)))}$$

(VI.8)

$$\frac{\partial\Pi^{M}}{\partial b}=r(c(b))[-D(p(w,b))+\frac{D(p(w,b))^{2}(w-cm+(\Delta-b)\cdot r(c(b)))}{X\cdot(-2D(p(w,b))+X^{2}\cdot D''(p(w,b)))}$$
$$+\frac{(b-cm+cr)\cdot r(c(b))D(p(w,b))}{-2(b-c(b))\cdot r(c(b))+(b-c(b))^{3}\cdot r''(c(b))}] \qquad \text{(VI.9)}$$

where $X=w-p(w,b)+(c(b)-b)r(c(b))$ Notice that $\partial\prod^{M}/\partial w$ is contained as a term in $\partial\prod^{M}/\partial b$. Set $\partial\prod^{M}/\partial w=0$ and $\partial\prod^{M}/\partial b=0$, we can get that

$$(b-cm+cr)\cdot r(c(b))D(p(w,b))=0 \qquad \text{(VI.10)}$$

However $D(p(w,b))=0$ means that demand equals to 0 and $r(c(b))=0$ means degenerating the problem to the supply chain model with forward flow only; we consider both cases are infeasible (i.e., the two solutions out of the multiple solutions are infeasible to the problem). Therefore we conclude that the only feasible solution of Scenario M is when $b^{*}=cm-cr=\Delta$ Here we also assume that the second order sufficient conditions at $b^{*}=\Delta$ are satisfied to ensure the global optimum. ■

APPENDIX VII. Proof of b_R^*<Δ=cm-cr in Scenario R

In Scenario R, given the retailer's announcement of β and λ, the manufacturer's profit is $\Pi_R^M(w,b) = D(\beta w)[w - cm + r(b\lambda)(\Delta - b)]$. We can find that the optimal b is independent on w. For any given w and $D(\beta w) > 0$ (feasibility requirement), we can find that $\Pi_R^M(w, b < \Delta) < \Pi_R^M(w, b \geq \Delta)$ because $r(b\lambda)$ >0 for nonzero b and λ (feasibility requirement). Therefore, the optimal b_R^* <Δ=*cm-cr* in Scenario R ■

APPENDIX VIII. Optimal Prices and Profits in Scenario M

From Proposition 1, we know that $b_M^* = \Delta$. We now consider the other optimal prices in the supply chain in Scenario M. From Proposition 1 ($b_M^* = \Delta$) and the fact that optimal collecting price c_M^* is independent on retail price p (considering $\prod^R = D(p)[p - w + r(c)(b - c)]$), we can derive that $c_M^* = \arg\underset{c}{\text{Max}}\, r(c)(\Delta - c)$ and thus $c_M^* = c_C^*$, i.e., the retailer will set the same collecting price c in Scenario C and Scenario M. Regarding the optimal wholesale price, we can find that $w_M^* > cm$, because if $w_M^* \leq cm$, then the manufacturer's profit $\prod^M = D(p)\cdot[w - cm + r(c)\cdot(\Delta - b)]$ is non-positive (considering $b_M^* = \Delta$), which is infeasible in the manufacturer-driven supply chain where no coordination mechanism exists. Last, the optimal retail price p_M^* of Scenario M can be found as $p_M^* = \arg\underset{p}{\text{Max}}\, D(p)[p - w_M^* + r(c_M^*)(\Delta - c_M^*)]$ and satisfies that $p_M^* \neq p_C^*$ We note here that $p_M^* > p_C^*$ when the condition $2D'(p)^2 > D(p)D''(p)$ is satisfied.

Considering $w_M^* \neq cm$ and $b_M^* = \Delta$ in Scenario M, we can derive that the total supply chain profit of Scenario M is less than that of Scenario C, i.e., $\Pi_M^T < \Pi_C^T$. In other words, the supply chain in Scenario M as a system is less efficient or has more inefficiency than the supply chain in Scenario C. However, since $c_M^* = c_C^*$ (i.e., the reverse flow behaves the same system-wise in Scenario M and Scenario C) while $p_M^* \neq p_C^*$ (i.e., the forward flow behaves differently system-wise), the reverse flow in Scenario M actually causes no inefficiency and the supply chain inefficiency in Scenario M is entirely caused by the forward flow (The "inefficiency" in our model means the total supply chain profit loss in Scenario M and Scenario R compared to Scenario C).

APPENDIX IX. Proof of (w=cm, b=Δ) to be a necessary condition for $\prod^T$ maximization in Scenario M

To proof (w=cm, b=Δ) is a necessary condition for $\prod^T$ maximization in Scenario M is equivalent to proof that if $b \neq \Delta$ or $w \neq cm$, then $p \neq p_C^*$ or $c \neq c_C^*$. We proof it as two steps:

(a) We first proof that no matter w=cm or $w \neq cm$, if $b \neq \Delta$ then $c \neq c_C^*$. Consider $\prod^R$=$D(p)$ [p-w+$r(c)(b$-$c)$]. Given w and b, the optimal c^* maximizing $\prod^R$ is independent on p and satisfies FONC $r'(c^*)(b\text{-}c^*)\text{-}r(c^*)=0$, or

$$b = r(c^*)/r'(c^*) + c^* \qquad \text{(IX.1)}$$

(IX.1) shows that b and c^* have one-to-one mapping relationship. We also know that if b=Δ then c=c_C^* (because of single optimum); therefore, we can find from (IX.1) that that if c=c_C^* then b=Δ, or if $b \neq \Delta$ then $c \neq c_C^*$.

(b) We then proof that under b=Δ, if $w \neq cm$ then $p \neq p_C^*$. Under b=Δ, $c = c_C^*$ and $\prod^R$= $D(p)[p$-w+$r(c)(b$-$c)]$ =$D(p)[p$-w+$r(c_C^*)(\Delta$- $c_C^*)]$. The optimal p^* which maximizes $\prod^R$ satisfies FONC $D'(p^*)[p^*\text{-}w+r(c_C^*)(\Delta\text{-} c_C^*)]+D(p^*)=0$ or

$$w = p^* + r(c_C^*)(\Delta - c_C^*) + D(p^*)/D'(p^*) \qquad \text{(IX.2)}$$

(IX.2) shows that w and p^* have one-to-one mapping relationship. Since (w=cm, b=Δ) is a sufficient condition for $\prod^T$ maximization (p= p_C^*, c=c_C^*), (IX.2) can be written as $cm = p_C^* + r(c_C^*)(\Delta - c_C^*) + D(p_C^*)/D'(p_C^*)$, which in turn means under b=Δ, if p= p_C^* then w=cm, or if $w \neq cm$ then $p \neq p_C^*$.

Considering (a) and (b), we can conclude that if $b \neq \Delta$ or $w \neq cm$, then $p \neq p_C^*$ or $c \neq c_C^*$, i.e., (w=cm, b=Δ) is a necessary condition for $\prod^T$ maximization (p= p_C^*, c=c_C^*). ■

APPENDIX X. Proof of (β=1, λ=1) to be a necessary condition for $\prod^{T}$ maximization in Scenario R

To proof (β=1, λ=1) is a necessary condition for $\prod^{T}$ maximization in Scenario R is equivalent to proof that if β≠1 or λ≠1, then $p \neq p_C^*$ or $c \neq c_C^*$. We proof it as two steps:

(a) We first proof that no matter β=1 or β≠1, if λ≠1 then $c \neq c_C^*$. Consider $\prod^{M}=D(p)$ [w-cm+r(c)(Δ-b)]= D($w\beta$) [w-cm+r(λb)(Δ-b)]. Given β and λ, we find that the optimal b^* maximizing $\prod^{M}$ is independent on w and satisfies FONC $r'(\lambda b^*)\lambda(\Delta\text{-}b^*)\text{-}r(\lambda b^*)=0$, or

$$\lambda = r(c^*)/[r'(c^*)\Delta] + c^*/\Delta \qquad (X.1)$$

where $c^*=\lambda b^*$. (X.1) shows that λ and c^* have one-to-one mapping relationship. When λ=1, we find that $c=b=c_C^*$, and (X.1) can be written as $\lambda = 1 = r(c_C^*)/[r'(c_C^*)\Delta] + c_C^*/\Delta$, which in turn means if $c = c_C^*$ then λ=1, or if λ≠1 then $c \neq c_C^*$.

(b) We then proof that under λ=1, if β≠1 then $p \neq p_C^*$. Under λ=1, $b = c = c_C^*$ and $\prod^{M}=D(p)$ [w-cm+r(c)(Δ-b)]= D($w\beta$) [w-cm+r(c_C^*)(Δ- c_C^*)]. The optimal w^* which maximizes $\prod^{M}$ satisfies FONC $D'(w\beta)\beta[w\text{-}cm+r(c_C^*)(\Delta\text{-}c_C^*)]+D(w\beta)=0$ or

$$\beta = [D(p^*)/D'(p^*) + p^*]/[cm - r(c_C^*)(\Delta - c_C^*)] \qquad (X.2)$$

where $p^*=\beta w^*$ (X.2) shows that β and p^* have one-to-one mapping relationship. Since (β=1, λ=1) is a sufficient condition for $\prod^{T}$ maximization ($p=p_C^*$, $c=c_C^*$), (X.2) can be written as $\beta = 1 = [D(p_C^*)/D'(p_C^*) + p_C^*]/[cm - r(c_C^*)(\Delta - c_C^*)]$, which in turn means under λ=1, if $p=p_C^*$ then β=1, or if β≠1 then $p \neq p_C^*$.

Considering (a) and (b), we can conclude that if β≠1 or λ≠1, then $p \neq p_C^*$ or $c \neq c_C^*$., i.e., (β=1, λ=1) is a necessary condition for $\prod^{T}$ maximization ($p=p_C^*$, $c=c_C^*$). ■

APPENDIX XI. Retailer-driven model with forward flow only

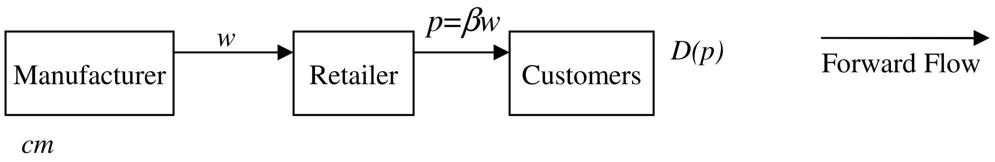


Figure 15. Retailer-driven Model with Forward Flow Only

In the retailer-driven model with forward flow only (see Figure 15), the manufacturer's profit is $\Pi^M = D(p)(w-cm) = D(w\beta)(w-cm)$; the retailer's profit is $\Pi^R = D(p)(p-w) = D(w\beta)(w\beta - w)$.

Consider the linear demand function form, i.e., $D(p) = u - vp$. By solving $\partial\Pi^M / \partial w = 0$, we find the manufacturer's reaction function is $w(\beta) = u/(2v\beta) + cm/2$. By taking into account the manufacturer's reaction function, the retailer's profit is

$$\Pi^R = \frac{(\beta-1)(u^2 - v^2\beta^2 cm^2)}{4v\beta}$$

We could solve the FONC $\partial\Pi^R / \partial\beta = 0$ and then check the SOSC of the FONC satisfying point to determine the optimal solution of the problem.

Using the same parameter values in the numerical example in Section 4.1, i.e., u=1000, v=20, cm=5, we find that retailer's problem in the retailer-driven model with forward flow only is

$$\Pi^R = \frac{(\beta-1)(u^2 - v^2\beta^2 cm^2)}{4v\beta} = \frac{125(\beta-1)(100-\beta^2)}{\beta}$$

By solving the FONC and checking the SOSC, we find the optimal $\beta^* = 3.858$. The retailer's profit is Π^R=7881.71.

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